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MATH 20D - Practice Exam #1
The exam covers Sections 1.1-3.2 (inclusive) minus 2.8 and 2.9
closed book, no calculators, no computers, no notes, no headphones ...
each problem is worth the same number of points

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1) a) Find an integrating factor u for $y' - xy = -x$.
   b) Solve the ODE in part a) by multiplying it by the integrating factor u and recognizing the left hand side as $(uy)'$.

2) Solve the separable ODE $y\,y' + x = 0$ with initial condition $y(2)=2$. Find the interval $a \le x \le b$ where the solution $y(x)$ is defined. If you were to plot the direction field, what would you expect to see?

3) A tank contains 100 gallons of salty water made by dissolving 80 lb. of salt in water. Pure water runs into the tank at the rate of 4 gal./min, and the well-stirred mixture runs out at the same rate. Let $Q(t)$ be the amount of salt in the tank at time t.
   a) Set up the ODE for this situation using $Q'(t)$ = rate in - rate out.
   b) Find the formula for $Q(t)$.
   c) Find the time required for half the salt to leave the tank. Don't compute the decimal approximation here.

4) True - False. Tell whether the following statements are true or false. Give a brief reason for your answer.

   a) The ODE $(y \cos x + 2x e^y) + (\sin x + x^2 e^y - 1)\, y'(x) = 0$ is exact.

   b) If f and $\dfrac{\partial f}{\partial y}$ are continuous in a rectangle $|x-x_0| \le a$, $|y-y_0| \le b$,
   then there is some interval $|x-x_0| \le h$ on which the initial value problem
   $$y'(x) = f(x,y), \quad f(x_0) = y_0,$$
   cannot have 2 distinct solutions.

   c) The logistic equation $y'(x) = ry(1-y/K)$ has two equilibrium solutions $y = K$ and $y = 0$. The first of these is stable while the second is unstable. Assume that the constants r and K are both positive.

5) a) Find a fundamental set of solutions for $y'' + y' - 2y = 0$.

   b) Compute the Wronskian determinant of the fundamental set from part a).

   c) Solve the initial value problem: $y'' + y' - 2y = 0$, $y(0)=2$, $y'(0) = 3$.

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