1) Suppose that $R$ is a ring with 1, an identity for multiplication.
   a) Show that this identity for multiplication is unique.
   b) Show $(-1)a = -a$ for all $a$ in $R$.
   c) Show that $(-1)(-1) = 1$.

2) Check that the set $C(\mathbb{R})$ consisting of all continuous real valued functions on the real line forms a commutative ring if you define $(f+g)(x) = f(x) + g(x)$ for all $x$ in $\mathbb{R}$, and $(fg)(x) = f(x)g(x)$, $\forall x \in \mathbb{R}$. Here we assume that $f, g$ are in $C(\mathbb{R})$. Does this ring have an identity for multiplication?

3) Let $\mathbb{R}[x]$ denote the ring of polynomials with real coefficients; i.e. elements of the ring have the form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, where the coefficients $a_j$ are real numbers. Define the sum of polynomials as in Part 4 of the Lectures on p. 6. Find the group of units:
   $$U(\mathbb{R}[x]) = \{ p \in \mathbb{R}[x] \mid p^{-1} \in \mathbb{R}[x] \}.$$  
   Hint: Recall what happened in part 4 of the Lecture Notes when $\mathbb{R}$ is replaced by $\mathbb{Z}$.

4) a) Show that $2\mathbb{Z} \cup 5\mathbb{Z}$ is not a subring of $\mathbb{Z}$.
   b) Show that $2\mathbb{Z} + 5\mathbb{Z} = \{ a+b \mid a \in 2\mathbb{Z} \text{ and } b \in 5\mathbb{Z} \} = \mathbb{Z}$.
   c) Show that $2\mathbb{Z} \cap 5\mathbb{Z} = 10\mathbb{Z}$.

5) Consider the ring
   $$R = \left\{ \begin{pmatrix} a & a \\ b & b \end{pmatrix} \mid a, b \in \mathbb{Z} \right\}.$$  
   Assume addition is componentwise and multiplication is the usual matrix multiplication. Prove or disprove that $R$ is a subring of the ring $M_2(\mathbb{Z})$ of all 2x2 matrices with integer entries.