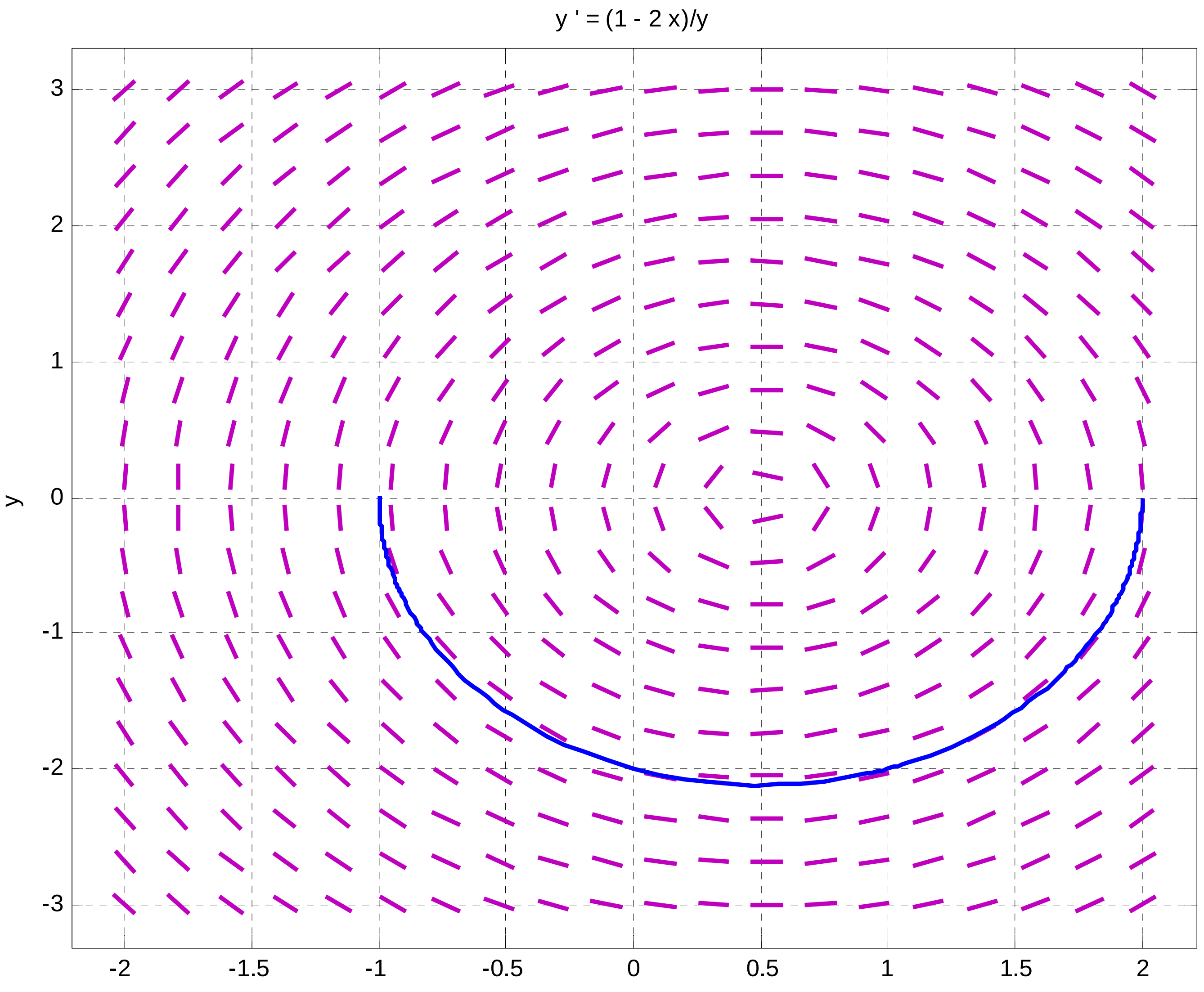
A SEPARABLE ODE

$$y' = (1-2x)/y$$

$$y(1) = -2$$



$dy/dx = (1-2x)/y$; $y(1) = -2$

Put all the y's on one side and all the x's on the other.   Then find the interval on the x-axis where the solution is defined.

$$y\,dy = (1-2x)\,dx$$

Integrate

$$\int y\,dy = \int (1-2x)\,dx$$

$$\tfrac{1}{2} y^2 = x - x^2 + C$$

$$y^2 = 2x - 2x^2 + C'$$

Plug $x=1$ and $y=-2$ to find C'. Obtain $C'=4$.

$$y = -\sqrt{-2x^2 + 2x + 4}$$

We choose the minus sign thanks to the initial condition.

$$y = -\sqrt{-2x^2 + 2x + 4}$$

Where does this make sense? You can't take the square root of negative numbers if you want a real answer.

This means we need

$-2x^2 + 2x + 4 \geq 0$

$-x^2 + x + 2 \geq 0$

$x^2 - x + 2 \leq 0$

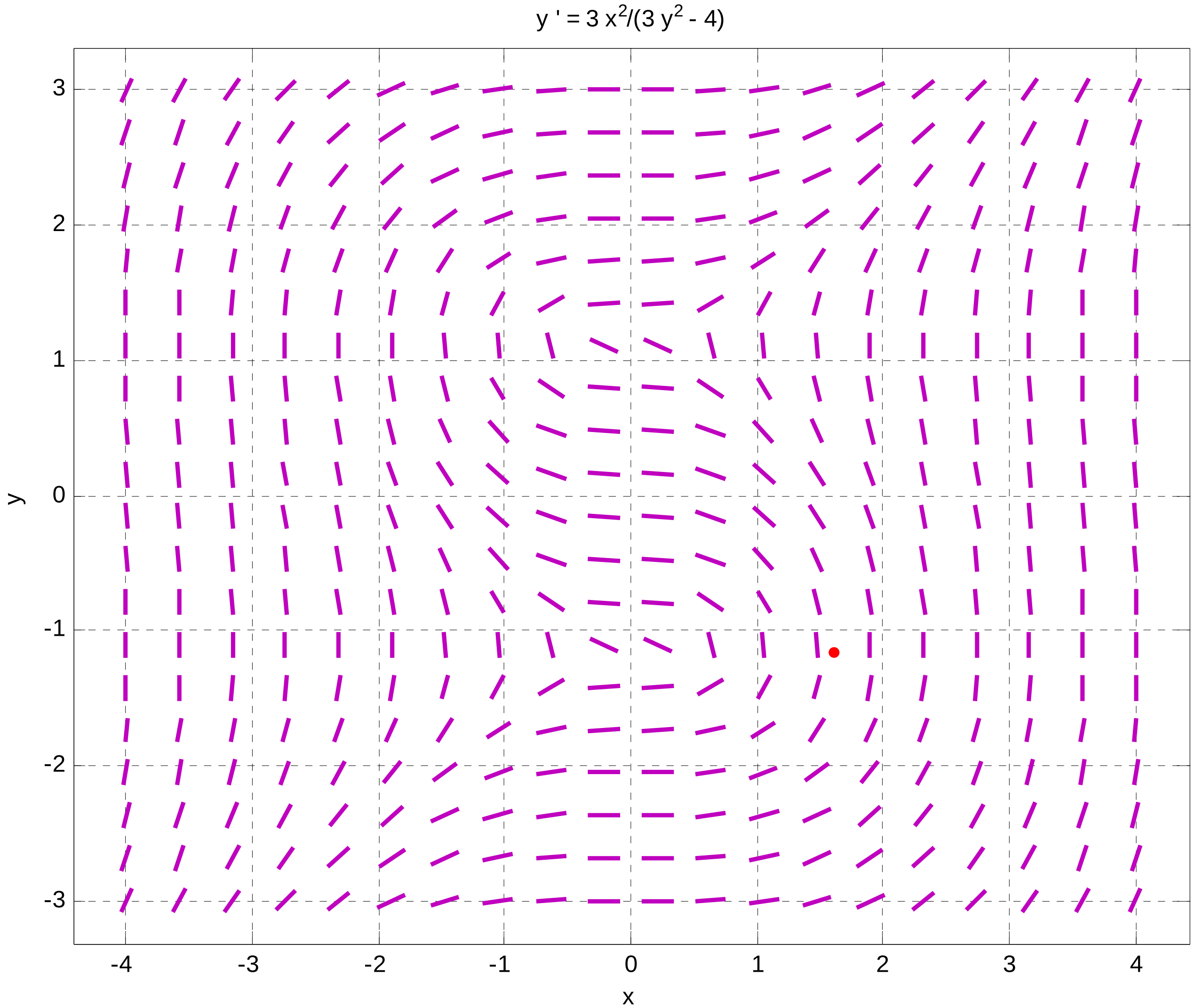
$x^2 - x + 2 = (x-2)(x+1) \leq 0$

This only happens if $-1 \leq x \leq 2$.

The endpoints correspond to $y=0$ where the differential equation $y' = (1-2x)/y$

does not make sense.

Another Example: $y' = 3x^2/(3y^2-4)$; $y(1)=0$



Matlab behaved badly and bombed my computer when I tried to get it to draw the numerical solution curve thru (1,0). Non-linear problems cause Matlab trouble.

Still we can find out much about the solution, although not a very good formula.

$(3y^2-4)dy = \quad 3x^2\, dx$

Integrate to get $\quad y^3-4y = x^3 + C.$

$y(1)=0$ implies $C=-1$.

Our "Solution" is: $\quad y^3-4y = x^3 + -1$

Unfortunately this does not lead to a simple formula for y as a function of x. But as one class member said: " You can easily solve for x."

Mathematica will give a very complicated answer with many roots inside of roots including some complex roots when you solve for y as we are asked to do in Section 2.2.

To figure out what values of x are legal, look back at the differential equation.

$y' = 3x^2/(3y^2-4)$

Where is the denominator =0?

Answer: Where $y^2=4/3$

$y = \pm 2/\sqrt{3}$

For this value of y you can solve for x using our equation

$y^3-4y = x^3 + -1$.

We see that $y^3-4y +1 = x^3$.

Thus $x = (y^3-4y +1)^{1/3}$.

Plug $y = \pm 2/\sqrt{3}$ into this and see that the legal x-values lie between -1.276 and 1.598.

Here's the direction field and a solution starting at the point x=1, y=0, drawn in blue.

It does seem to stretch from -1.3 to 1.6, where it encounters vertical tangent lines that send y(x) off to infinity. The red point is the only thing produced by Matlab when asked for a solution thru (1,0).