Math 163 Sample midterm solutions

1) 26 = ||||||∩∩

75 = |||||∩∩∩∩ ∩∩∩

26 · 75 :

	|||||∩∩∩∩ ∩∩∩
✓		∩∩∩ ∩∩ 9
				999
✓ ||||||	999999
✓ ||||||∩	99
||||||∩∩	∩∩∩ 99999 ∩∩ 9999

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2) a) Area of circle $\approx (d - \frac{1}{9}d)^2 = (18-2)^2 = 256$ cubits^2

b) $A = \pi r^2 \approx 256$ cubits^2 $r = \frac{d}{2} = 9$ cubits, so $81\pi \approx 256$

$\pi \approx \frac{256}{81} = \left(\frac{16}{9}\right)^2 = 4\left(\frac{8}{9}\right)^2$

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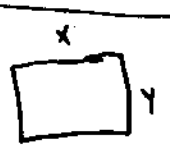
3) Otto Neugebauer: F, V
Pythagoras of Samos: B, II
Jean Francois Champollion: D, IV
Thales of Miletus: C, III
Henry Creswicke Rawlinson: E, VI
Almes: A, I

4)
$$\begin{array}{r} \phi \quad \upsilon \\ \chi \mu \eta \\ +\ \upsilon\ \pi\ \theta \\ \hline \alpha\ \rho\ \lambda\ \zeta \end{array}$$

5)



$x - y = 8$

$xy = 84$

(a) $(x+y)^2 = (x-y)^2 + 4xy = 64 + 4(84) = 400$

so $x + y = 20$

(b) $x = \frac{20}{2} + a = 10 + a$, $y = 10 - a$ since $\frac{x+y}{2} = 10$ is the average

(c) $84 = xy = (10+a)(10-a) = 100 - a^2$

$a^2 = 16$

$a = 4$

(d) $x = 10 + 4 = 14$

$y = 10 - 4 = 6$

6) The line: 65 97 5

The meaning: 65 and 97 are b and c of some Pythagorean triple. (5 is a line number)

$a = c^2 - b^2 = 97^2 - 65^2$

$97^2 = 9409$

$-65^2 = -4225$

$5184 = 72^2$

conclude $a = 72$, and $72^2 + 65^2 = 97^2$

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7) Want primitive Pythagorean triples (x, y, z) with $x = 84$

Know: all integer triples are $(2st, s^2 - t^2, s^2 + t^2)$

Since our triples should be primitive, they cannot all be even (or else we could factor out a 2). Moreover, we must have 2 odds in order to have $x^2 + y^2 = z^2$.

Thus we must have $x = 2st = 84$ in exactly that role (so we will not be able to get $x = s^2 - t^2$ or $s^2 + t^2$)

Well $84 = 2 \cdot 2 \cdot 3 \cdot 7$, so we can have:

s	t	s^2	t^2
42	1	1764	1
21	2	441	4
14	3	392	9
7	6	49	36

Note we must have $s > t$, so this is it.

So we get: $(84, 1763, 1765)$

$(84, 437, 445)$

$(84, 383, 401)$

$(84, 13, 85)$

Disclaimer: I believe all of the above answers are correct. However, I make no assurances that they are. The best way to check is to do the problems yourself and compare work.

-Andy.