1 a) $897 = 4\cdot 208 + 65$

$208 = 3\cdot 65 + 13$

$65 = 5\cdot 13$

$\boxed{\gcd(897, 208) = 13}$

b) $\gcd(897,208) = 13$ and $13 \nmid 41$.

The linear diophantine equation $ax+by=c$ has a solution iff $d \mid c$, where $d = \gcd(a,b)$ (p 228), so there is no solution (over $\mathbb{Z}$).

c) $13 = 208 - 3\cdot 65 = 208 - 3(897 - 4\cdot 208)$

$= 5\cdot 208 - 3\cdot 897$

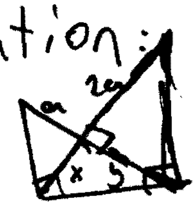
$39 = 3\cdot 13 = 3(5\cdot 208 - 3\cdot 897)$

$= 15\cdot 208 - 9\cdot 897$

$x = 15 \qquad y = -9$

2. a) The Delian problem was: given a cube of side a, find a ~~cube~~ length x so that $x^3 = 2a^3$ (doubling the volume of a cube).

b) One solution: construct a figure as below.



Using similar triangles, we get $\frac{a}{x} = \frac{x}{y} = \frac{y}{2a}$.

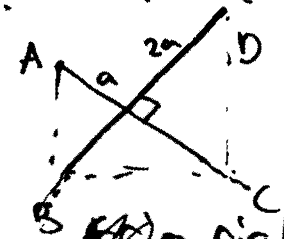
This gives us $ay = x^2$ and $2xa = y^2$, hence $2a^3 x = x^4$,

$\Downarrow$

$a^2y^2 = x^4$

so $2a^3 = x^3$.

c) At some point in this construction, you will need to measure angles. With a straight edge and compass, you can construct



but won't be able to construct the remainder of the ~~the~~ right triangles $\triangle ABC$ and $\triangle BCD$.

3. a) Four 'tacit assumptions' that Euclid makes are:
   (1) the existence of points, lines, etc.
   (2) uniqueness of a line segment joining two points
   (3) if a circle (or line) has one point inside and one point outside ~~a~~ another circle, then it intersects the circle in two points
   (4) some kind of congruence.

b) Four important things that contributed to the success of the Elements are:
   (1) the book gathers a large body of knowledge into a ~~well-organised~~ single body.
   (2) it is well organised; the ~~axioms~~ propositions are very ~~[illegible]~~ logically arranged
   (3) The axioms were wisely chosen so many propositions could be derived from few assumptions
   (4) The proofs were rigorous.

$\sqrt{3} \notin \mathbb{Q}$

4. Suppose to the contrary that there is a rational number $\frac{m}{n}$ such than $(\frac{m}{n})^2 = 3$.

Further, suppose without loss of generality that any common factors have been cancelled, so $\gcd(m,n)=1$.

$(\frac{m}{n})^2 = 3 \Rightarrow m^2 = 3n^2$, ① so $3 | m^2$.

Since 3 is prime, this implies $3|m$, (by the thm p180 or by cases as in homework5) so $m=3k$ for some integer k.

Equation ① then gives us $9k^2=3n^2$. This means $3k^2 = n^2$, i.e., $3|n^2$ so, again by the thm on p180, $3|n$. Sine $3|n$ and $3|m$, $\gcd(m,n) \geq 3$. ⨳.

Thus $\sqrt{3} \notin \mathbb{Q}$. $\square$

5 Proclus IV B

Eratosthenes III C

Euclid II D

Eudoxus I A

6 Euclid's lemma If $a|bc$ and $\gcd(a,b)=1$ then $a|c$.

Proof We take for granted that $\gcd(a,b)=1$ implies the existence of integers x & y so that $ax+by=1$.

The we can write

$$c = c\cdot 1 = c(ax+by) = acx + bcy = acx + aky = a(cx+ky).$$

($bc = ak$ for some k)

Thus $a|c$.

$\square$