Mathematics 163 Name: ______________________

January 20, 2010

SAMPLE MIDTERM EXAMINATION

Please use no notes and no books. Please use no calculators and no other electronic devices. Give complete answers; in particular, show calculations.

1. Express the numbers in the Ancient Egyptian notation. With that notation, carry out the multiplication by the method of the Ancient Egyptians. Translate your answer into our standard base ten system.

26 times 75

2. These questions relate to finding the area of a circle.

(a) Consider a circle of diameter 18 cubits. Use the Ancient Egyptian method to find the area the circle encloses. You may use modern notation for the numerals and operations.

(b) Recall that the area of a circle is πr^2. Find the rational approximation to π that the previous part suggests.

3. For each person, indicate the appropriate time period and contribution:

Otto Neugebauer	_________	__________
Pythagoras of Samos	_________	__________
Jean Francois Champollion	_________	__________
Thales of Miletus	_________	__________
Henry Creswicke Rawlinson	_________	__________
Ahmes	_________	__________

A. c 1620 B.C. B. c. 585 - 501 B.C. C. c. 625 – 547 B.C.

D. 1790 – 1832 E. 1810 – 1895 F. 1899 – 1990

I. Copied an older mathematical work

II. Founded a group that studied mathematics, among other activities

III. Made proofs part of mathematics

IV. First to translate the ancient Egyptian hieroglyphics

V. Made major contributions to the modern understanding of Babylonian mathematics in particular, Plimpton 322.

VI. Made major contributions to the modern ability to read ancient cuneiform writing

4. Carry out the following addition problem in the Ionic numeral system. (You may use the table on page 16.)

$$\chi\mu\eta'$$
$$\upsilon\pi\theta'$$

5. Recall the identity that $(x-y)^2 = (x+y)^2 - 4xy$. Solve the following problem in the Babylonian fashion as indicated in the remarks after the problem.

> The area of a rectangle of a rectangle is 84 square cubits. The difference between the sides of the rectangle is 8 cubits. Find the sides of the rectangle.

Use the identity to solve for the sum of the lengths of the sides of the rectangle.

Find a number such that the sides can be expressed as that number plus an unknown number and the number minus the same unknown number.

Find the unknown number from the given area of the rectangle.

Find the two sides of the rectangle.

6. Consider table on page 75 and its translation on page 73. Use the numbers on line 5 to illustrate the generally accepted interpretation of the table.

7. Find all the primitive Pythagorean triples x,y,z with $x = 84$.

(Hint: If $x = 84$, then $x = 2\cdot(2\cdot 7\cdot 3)\cdot(1)$ and $= 2\cdot(7\cdot 3)\cdot(2)$ and $= 2\cdot(7)\cdot(2\cdot 3)$ ... .)

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Theorems A&B were proved in class on Friday, January 15, 2010. cf. text page 295-7.

Th. A If s and t are positive integers with $s > t$, then the numbers

$x = 2st,\ y = s^2 - t^2,\ z = s^2 + t^2$ form a Pythagorean Triple, that is, $x^2 + y^2 = z^2$.

Th. B If s and t is a pair of integers with $s > t > 0$, and with no common divisor > 1 and with one of them even, then the numbers $x = 2st,\ y = s^2 - t^2,\ z = s^2 + t^2$ form a primitive Pythagorean Triple, that is, there is no common divisor d of $x, y,$ and z with d>1.

Theorem C (Page 297 of the text) If x,y,z form a primitive Pythagorean Triple, then there exists a pair of integers s,t such that they no common divisor $d > 1$, and one of them is even and $s > t > 0$, and the numbers $x = 2st,\ y = s^2 - t^2,\ z = s^2 + t^2$ form a primitive Pythagorean Triple.