

180A PRACTICE MIDTERM 1

I think these problems are representative, but you may find the actual midterm either more or less difficult than this practice midterm.

1. A card is dealt from a well-shuffled deck.
 - a. (5 points) What is the probability that it is a heart?
 $P(\heartsuit) = 13/52 = 1/4.$
 - b. (5 points) What is the probability that it is a face card?
 $P(F) = P(A \cup K \cup Q \cup J) = 4 \cdot 4/52 = 4/13.$
 - c. (5 points) What is the probability that it is a heart and a face card?
 $P(\heartsuit \cap F) = 4/52 = 1/13.$
 - d. (10 points) What is the probability that it is a heart or a face card?
 $P(\heartsuit \cup F) = P(\heartsuit) + P(F) - P(\heartsuit \cap F) = 1/4 + 4/13 - 1/13 = 25/52.$
2. (25 points) James Bond walks into the Monte Carlo casino in Monaco with €100,000. He plays roulette 10 times, wagering €10,000 on either red or black each time. What is the probability that he leaves the casino with at least €180,000?
[Hints: Monaco is in Europe. In European casinos there are 37 possible outcomes for a spin of the roulette wheel: 18 are red, 18 are black, and 1, the zero, is neither. The payoff odds on red, and on black, are 1 to 1.]
Let $p = 18/37$ be the probability of a win on any single spin of the roulette wheel. Leaving the casino with at least €180,000 means winning at least 9 times (don't forget to subtract €10,000 for each loss), and

$$P(\geq 9 \text{ wins}) = \binom{10}{9} p^9 (1-p) + \binom{10}{10} p^{10} = 10p^9(1-p) + p^{10}.$$

Working without a calculator, you may leave the answer like this.

3. (25 points) Tiger Woods hits his first shot at the par 4 first hole on Torrey Pines South into the rough, but decides to go for the green with his second shot. The probability that he puts the shot on the green is $7/12$ and the probability that it lands in one of the bunkers is $1/3$; with probability $1/12$ he hits it into the hole (this is Tiger we're talking about!). From a bunker, his probability of hitting the ball into the hole is $1/4$, while from the green his probability of hitting the ball into the hole is $1/2$. What is the probability that he scores a birdie on the hole?
[Hint: A birdie is one shot below par, so the question is, what is the probability that he hits the ball into the hole with exactly 2 more shots?]
To score a birdie Tiger must either hit the ball onto the green (G) and then into the hole (H), or into a bunker (S) and then into the hole. So $P(B) = P(H|G)P(G) + P(H|S)P(S) = (1/2)(7/12) + (1/4)(1/3) = 3/8.$
4. (25 points) What is de Finetti's definition for the probability of an event E ? Be precise. If someone is willing to pay pS to win S if E occurs, for any stake S , then that person is assigning probability p to E .