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MATH 180A. INTRODUCTION TO PROBABILITY LECTURE 25

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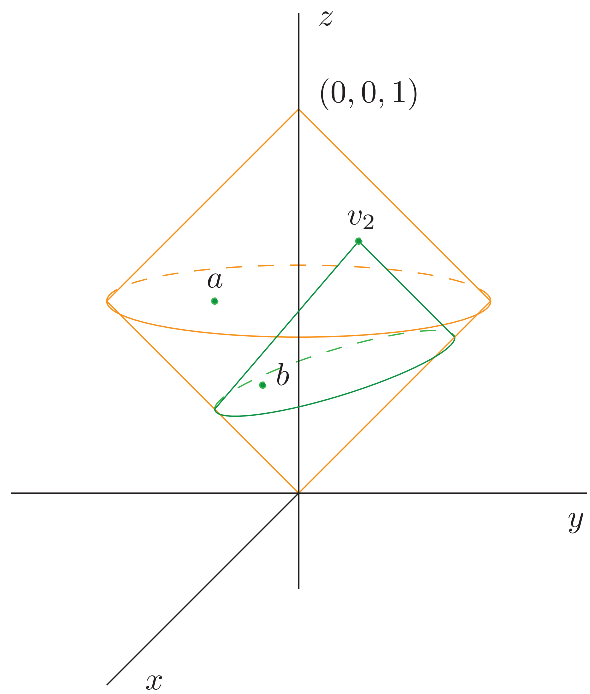
Extra Credit problem

Let V be the volume in $\mathbb{R}^3$ defined by the intersection of two solid cones,

$$\bigl\{(x,y,z) \mid z \geq \sqrt{x^2+y^2}\bigr\} \\ \cap \quad \bigl\{(x,y,z) \mid z \leq 1-\sqrt{x^2+y^2}\bigr\},$$

as shown in the figure. Let V_1 and V_2 be points picked independently and uniformly at random from V.



For v_1, $v_2 \in \mathbb{R}^3$, write $v_1 \prec v_2$ if v_1 is in the downward cone with apex at v_2, *i.e.*, if $(z_2-z_1)^2 \geq (x_1-x_2)^2+(y_1-y_2)^2$ and $z_1 \leq z_2$. In the figure, $a \not\prec v_2$ but $b \prec v_2$.

Find $P(V_1 \prec V_2)$.

Hint. Start by computing the volume of the intersection of the downward cone from $v_2 = (x,y,z)$ with V. You should find that it is a function only of $z^2-(x^2+y^2)$. As a warm-up you can do the two dimensional problem we did in class the same way.