180A MIDTERM 1

Please simplify your answers to the extent reasonable without a calculator. Show your work. Explain your answers, concisely.

1. [25 points] Let A, B, and C be events in a probability space $(\Omega, \mathcal{F}, P)$. Suppose $P(A) = P(B) = P(C) = 1/2$. What is the smallest possible value for $P(A \cap B) + P(B \cap C) + P(C \cap A)$?

2. A special *unfair* die has probabilities of rolling m and n whose ratio is m/n, for all $m, n \in \{1, 2, 3, 4, 5, 6\}$.
   a. [10 points] Find $P(n)$ for each $n \in \{1, 2, 3, 4, 5, 6\}$.
   b. [10 points] If you roll the die twice, what is the probability that the sum of your two rolls is 7?
   c. [5 points] Is your answer to (b) larger or smaller than what the probability would be if you were rolling a fair die?

3. [25 points] You play the following game with a fair die: Roll the die. If it is n, you roll the die n more times. If you roll a second n, you win. What is the probability that you win?

4. [25 points] Let $Z = (X, Y)$ be a point chosen uniformly at random in the unit square $[0, 1]^2 = \{(x, y) : 0 \leq x, y \leq 1\}$. Find the cumulative distribution function for the random variable $D =$ distance from Z to the closest point on the boundary of the square, and then find its probability density function.