

## 180A PRACTICE PROBLEMS FOR MIDTERM 1

Please simplify your answers to the extent reasonable without a calculator. Show your work. Explain your answers, concisely.

- Let  $A$  and  $B$  be events in a probability space  $(\Omega, \mathcal{F}, P)$ .
  - Suppose  $P(A) + P(B) > 1$ . Making no further assumptions on  $A$  and  $B$ , prove that  $A \cap B \neq \emptyset$ .
  - Suppose  $A \cap B = \emptyset$ . If  $A$  and  $B$  are independent, what can you say about  $P(A)$  and  $P(B)$ ?
  - Suppose  $P(A) = 1/2$  and  $P(B) = 4/5$ . What are the smallest and largest possible values for  $P(A \cap B)$ ?
- You are playing a game in which you roll 2 dice. If the sum of the two numbers showing is greater than or equal to 10, you win.
  - What is the probability that you win the first three times you play?
  - What is the probability that you win exactly three times out of the first five times you play?
  - What is the probability that the first game you win is before the tenth game, but after the fifth?
- A box contains three coins, two of which are fair and third of which has  $P(\text{heads}) = 3/4$ . A coin is chosen randomly from the box and flipped three times.
  - What is the probability that all three flips are heads?
  - Given that the three flips are all heads, what is the probability that the biased coin was chosen?
- Let  $P = (X, Y)$  be a point chosen uniformly at random in the unit square  $[0, 1]^2 = \{(x, y) : 0 \leq x, y \leq 1\}$ . Find the cumulative distribution function for the random variable  $Z = \min\{X, Y\}$ , and then find its probability density function.