Name: Section: **21D MIDTERM 2**

This is the *second* midterm from the last time I taught this class. That year the syllabus was organized so that we spent the first few weeks doing sequences and series, so the first midterm covered those topics. We have not yet completed studying second order differential equations in our class, so problem 4 is harder than what I will expect you to be able to do on our first midterm.

A formula that you might find useful on this test:

$$\big(D - a(t)\big)^{-1}[\cdot] = \frac{1}{\mu(t)} \int^{t} \mu(s)[\cdot]\mathrm{d}s, \text{ where } \mu(s) = e^{-\int a(s)\mathrm{d}s}.$$

1.a. (15 points) Find the general solution to the first order equation $ty' - 3y = t^4$.

b. (10 points) Find the solution that satisfies the initial condition $y(1) = 3$.

c. (10 points) Is there a solution that satisfies the initial condition $y(0) = 3$? Explain your answer, mentioning any relevant theorems that we have learned.

2.a. (10 points) Find the equilibrium solutions (that is, $y(t) = \textit{const.}$) to the first order equation $y' = y\ln(2/y)$. Determine whether each is asymptotically stable or unstable.

b. (10 points) Illustrate the graph $y(t)$ as a function of t for initial conditions $y(0) = y_0$, for $0 \le y_0 < \infty$.

c. (15 points) Solve this differential equation subject to the intial condition $y(0) = 1$. Hint: Use the substitution $u = \ln(2/y)$ when you integrate.

3.a. (10 points) Find the general solution to the second order equation $y'' + 2y' + 2y = 0$.

b. (10 points) Find the solution that satisfies the initial conditions $y(\pi/4) = 2$, $y'(\pi/4) = -2$.

c. (10 points) Describe the behavior of the solution satisfying these initial conditions as t goes to ∞.

4. (Extra credit: 25 points) Find the general solution to the second order differential equation $y'' - \frac{3}{t}y' + \frac{4}{t^2}y = 0$. Hint: Start by computing $\big(D - \frac{1}{t}\big)\big(D - \frac{2}{t}\big)[y]$.