Fibonacci numbers

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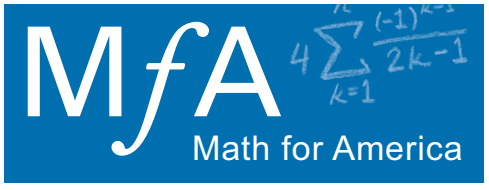
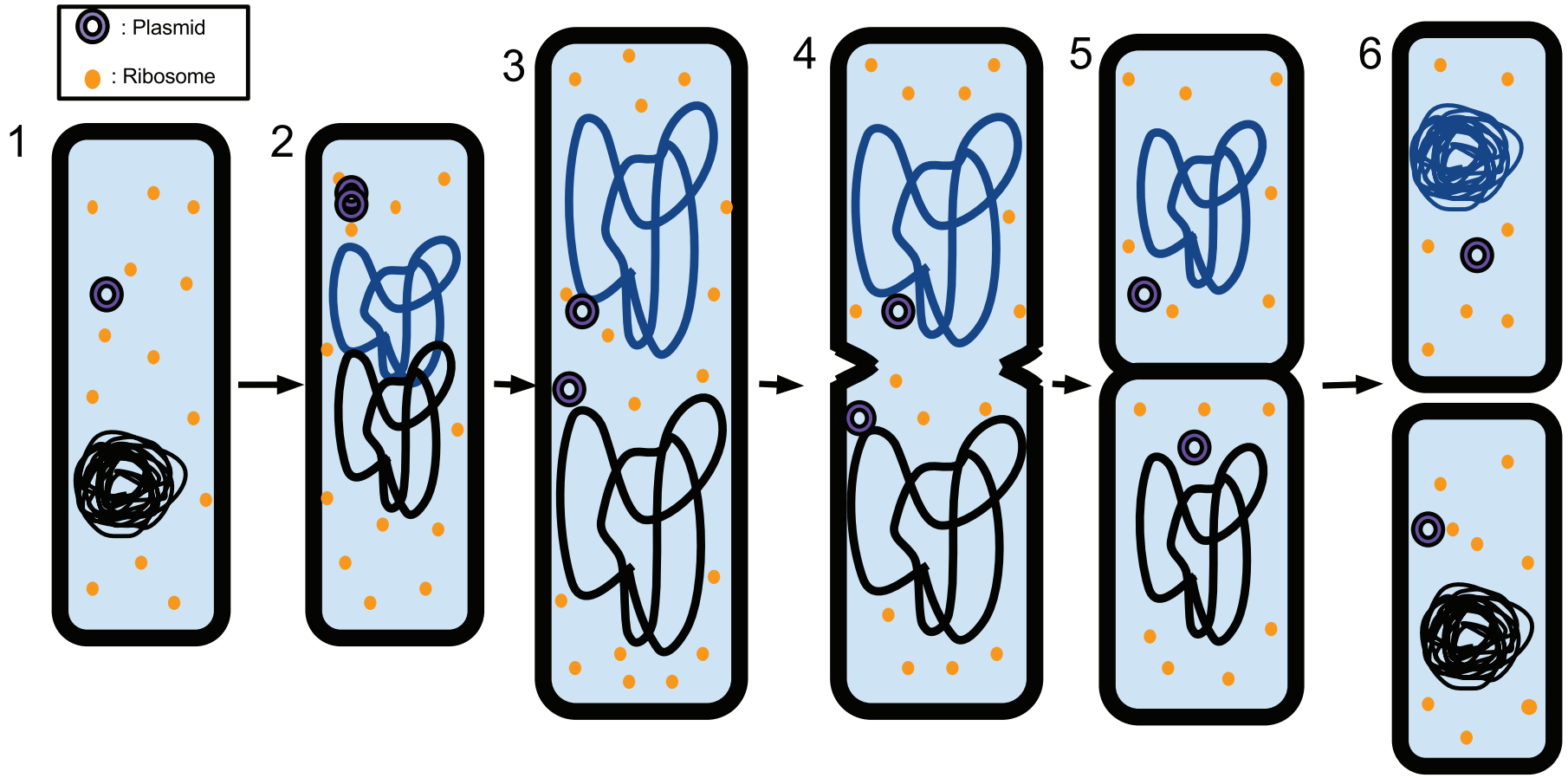
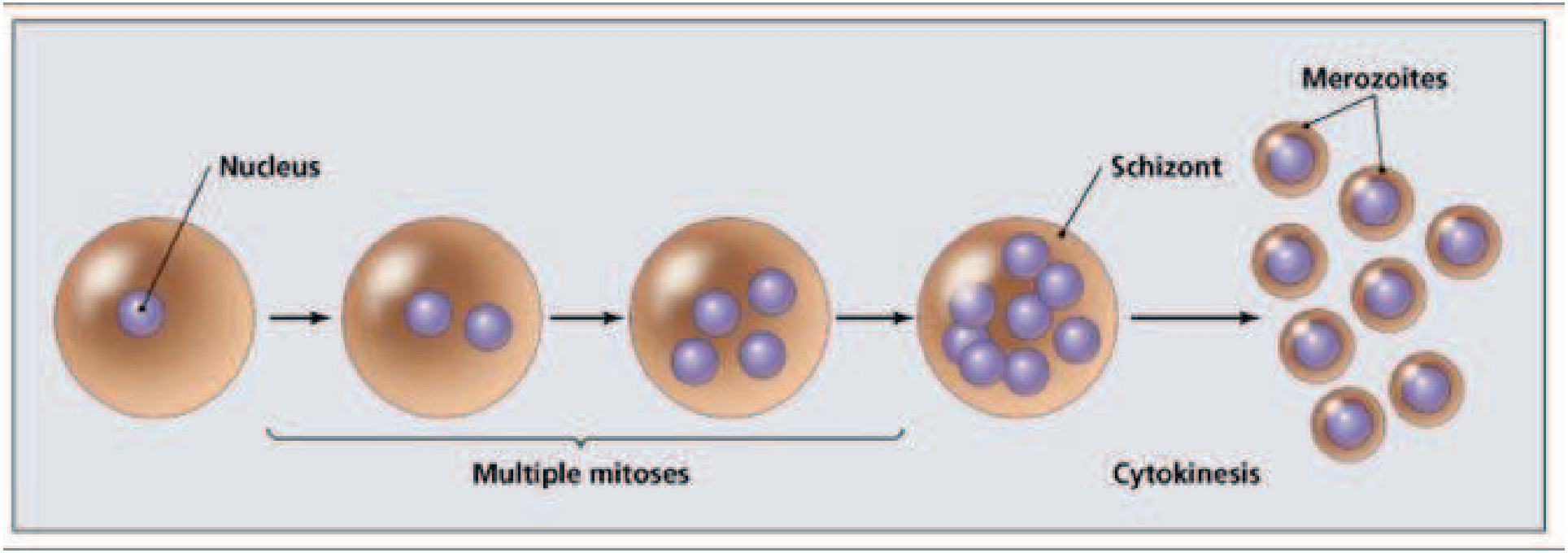


Binary fission



bacterial reproduction

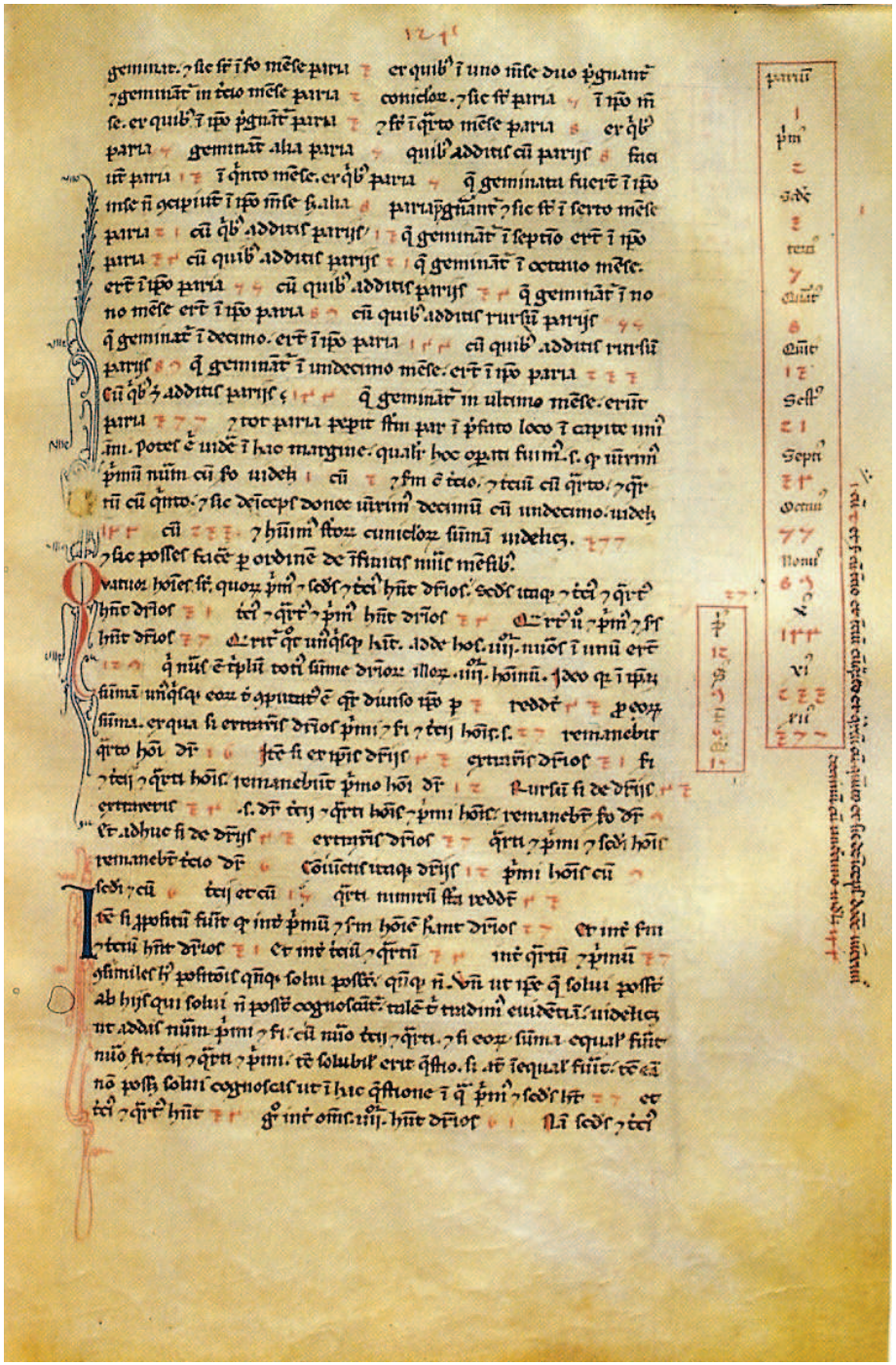
Multiple fission



parasite reproduction

Leonardo of Pisa — Fibonacci (c.1170–c.1250)

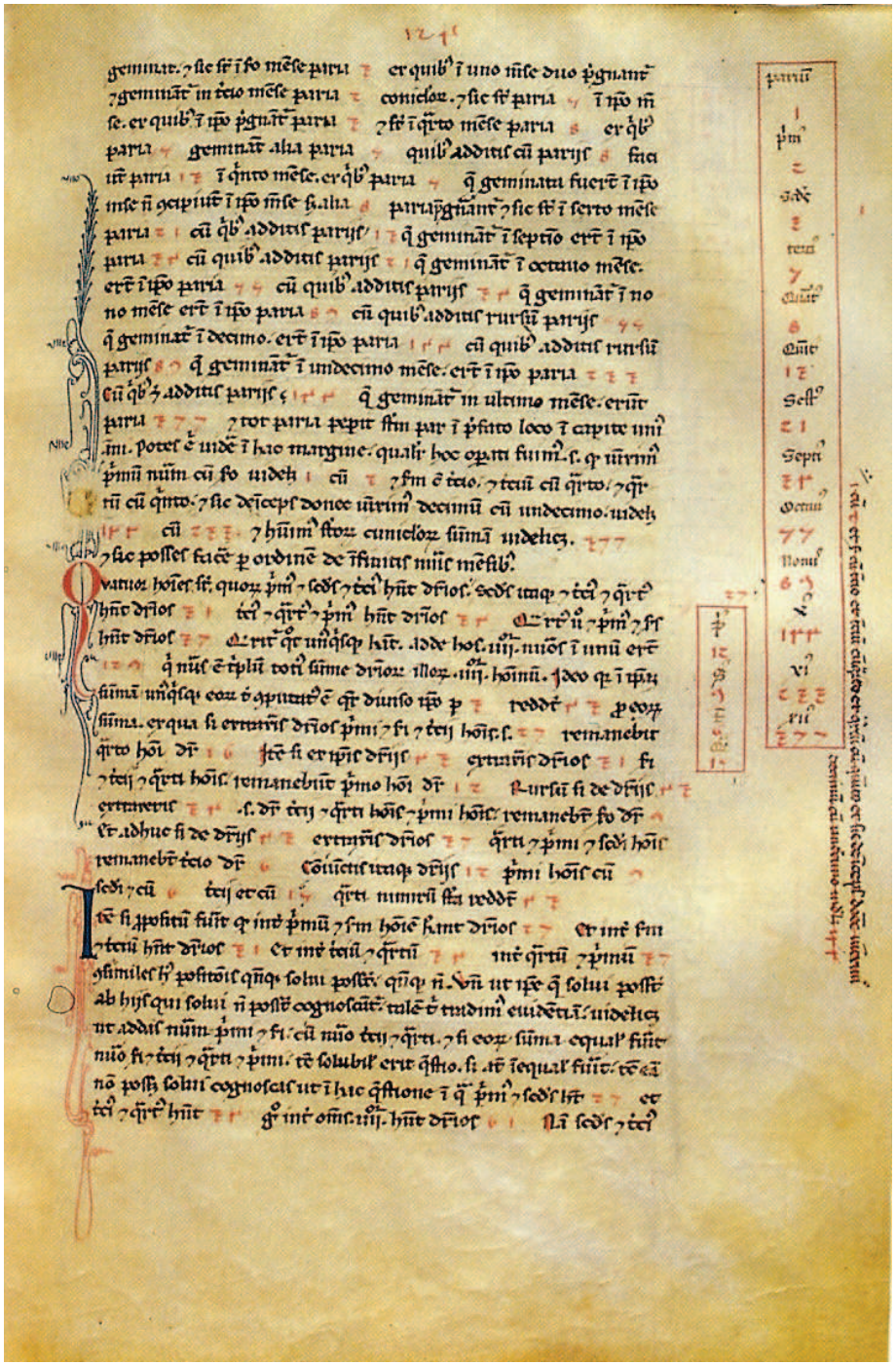
Liber Abaci (1202)



Liber Abaci (1202)

Quot paria coniculorum in uno anno ex uno pario germinentur.

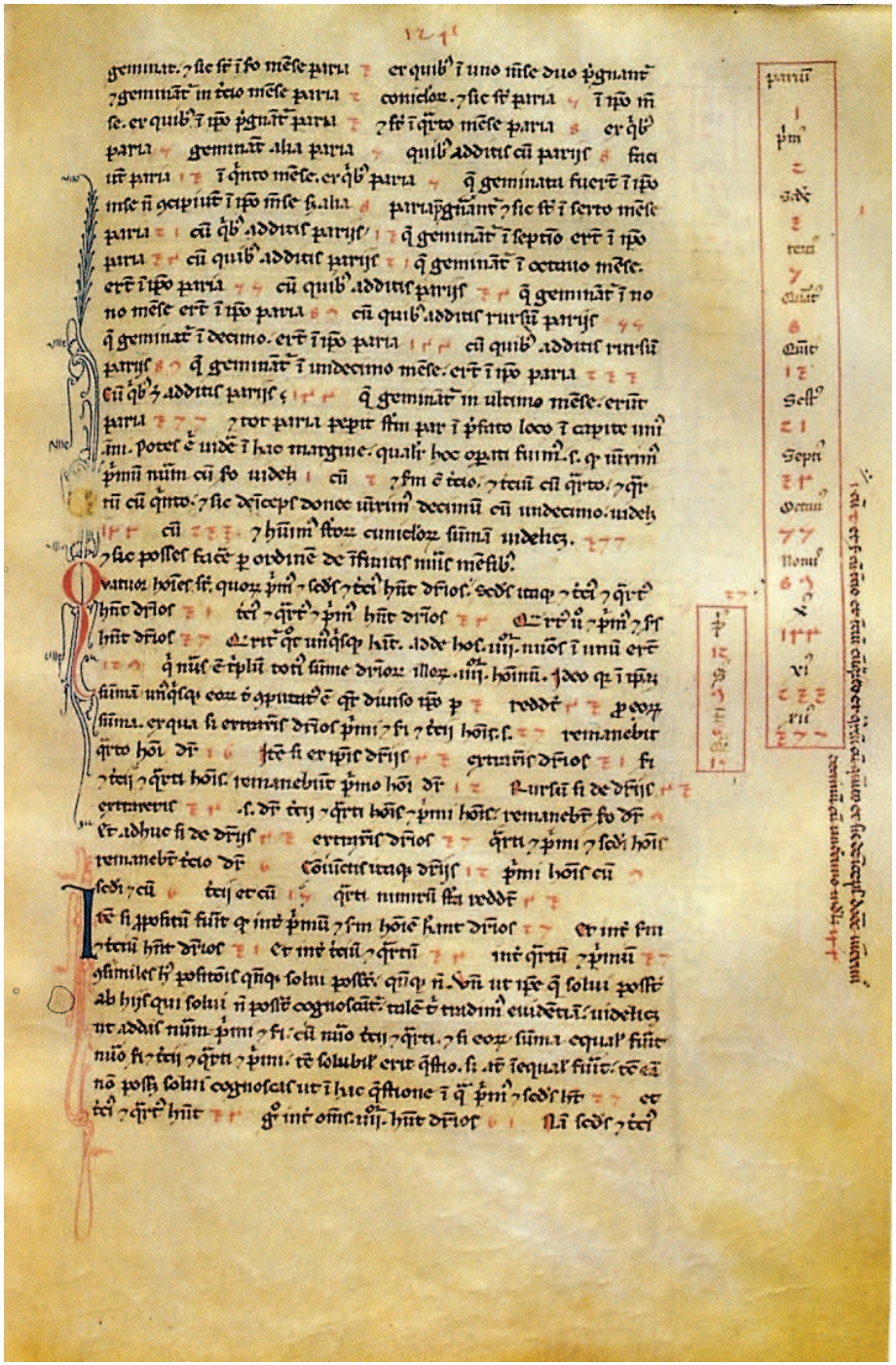
Quidam posuit unum par cuniculorum in quodam loco, qui erat undique pariete circundatus, ut sciret, quot ex eo paria germinarentur in uno anno, cum natura eorum sit per singulum mensem aliud par germinare, et in secundo mense ab eorum natiuitate germinant.



Liber Abaci (1202)

How many pairs of rabbits are created by one pair in one year?

A man placed one pair of rabbits together in an enclosure, so he might know how many are created from a pair in one year, when it is the nature of them in a single month to bear another pair, and in the second month those born to bear also.



Phyllotaxis

pinecone

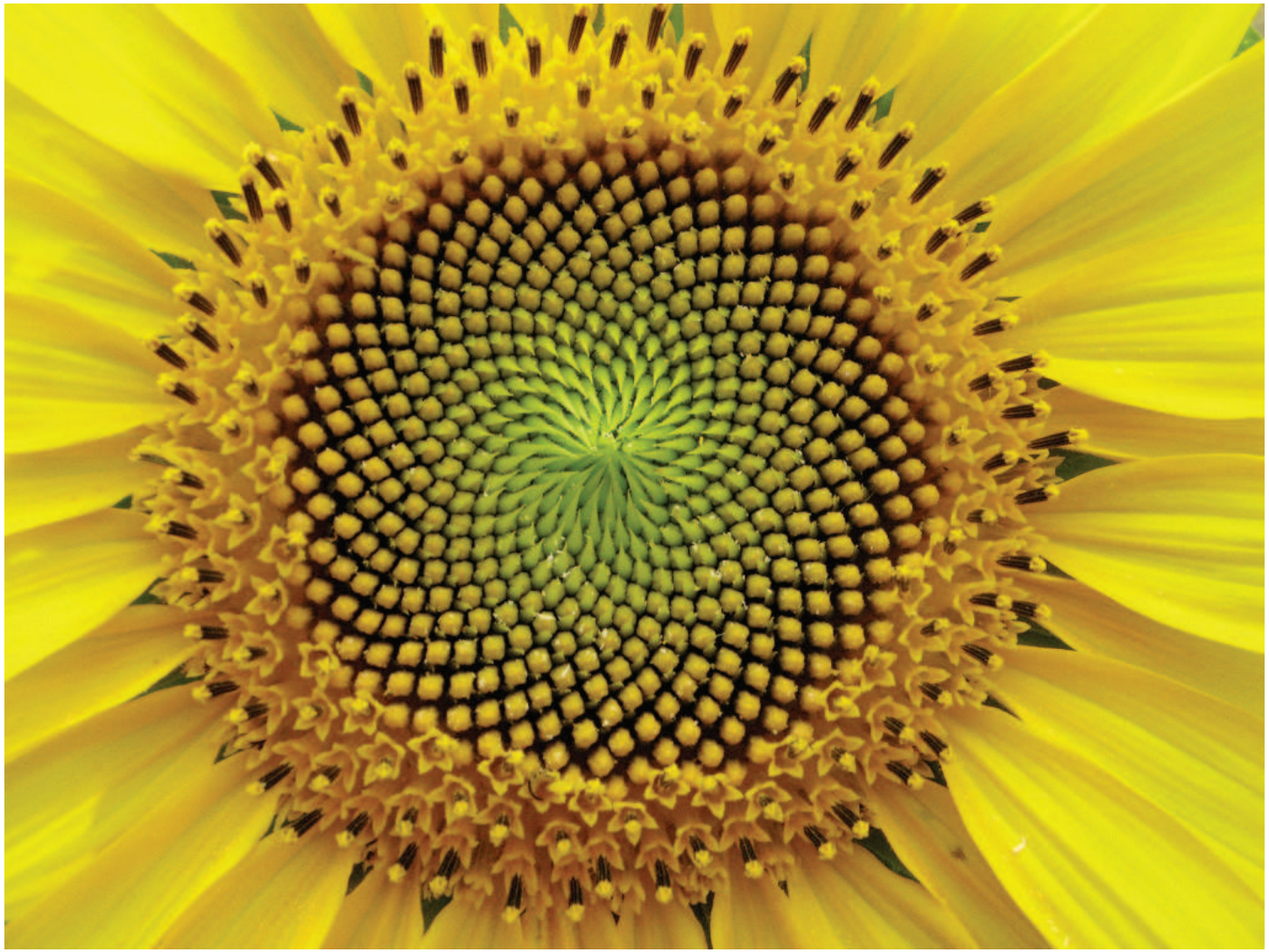
Phyllotaxis

8 spirals counterclockwise

Phyllotaxis

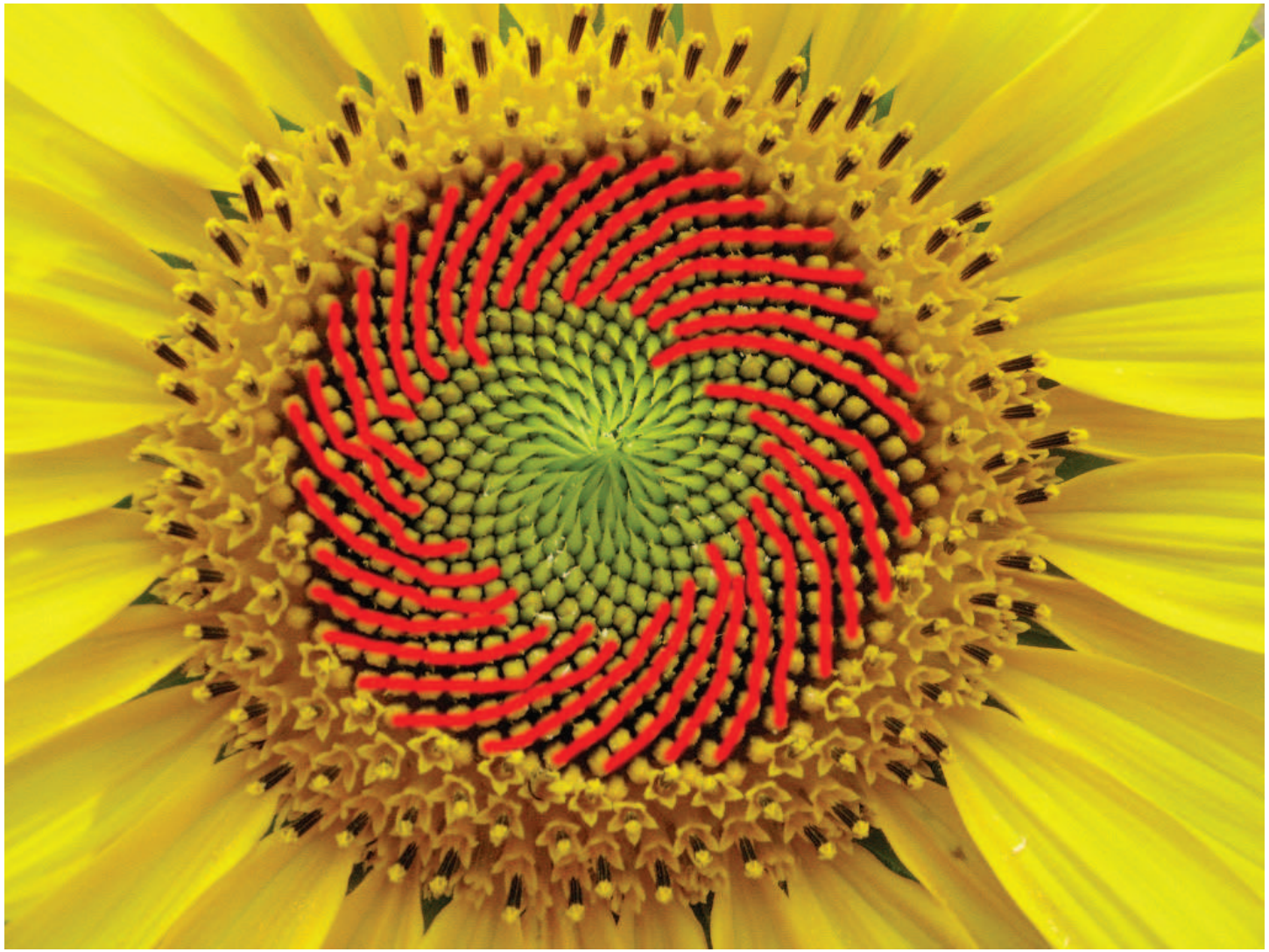
13 spirals clockwise

Phyllotaxis



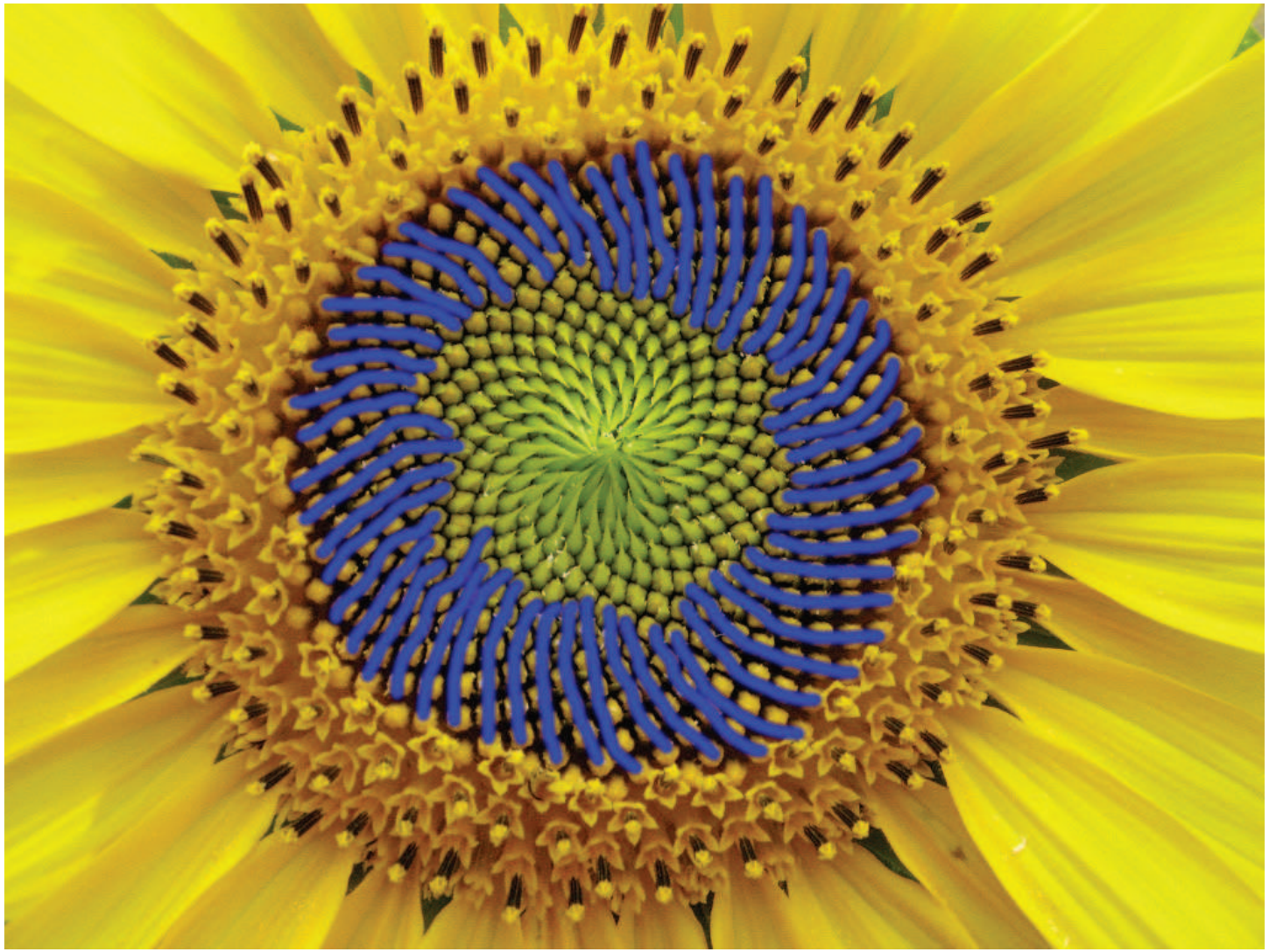
sunflower

Phyllotaxis



34 spirals clockwise

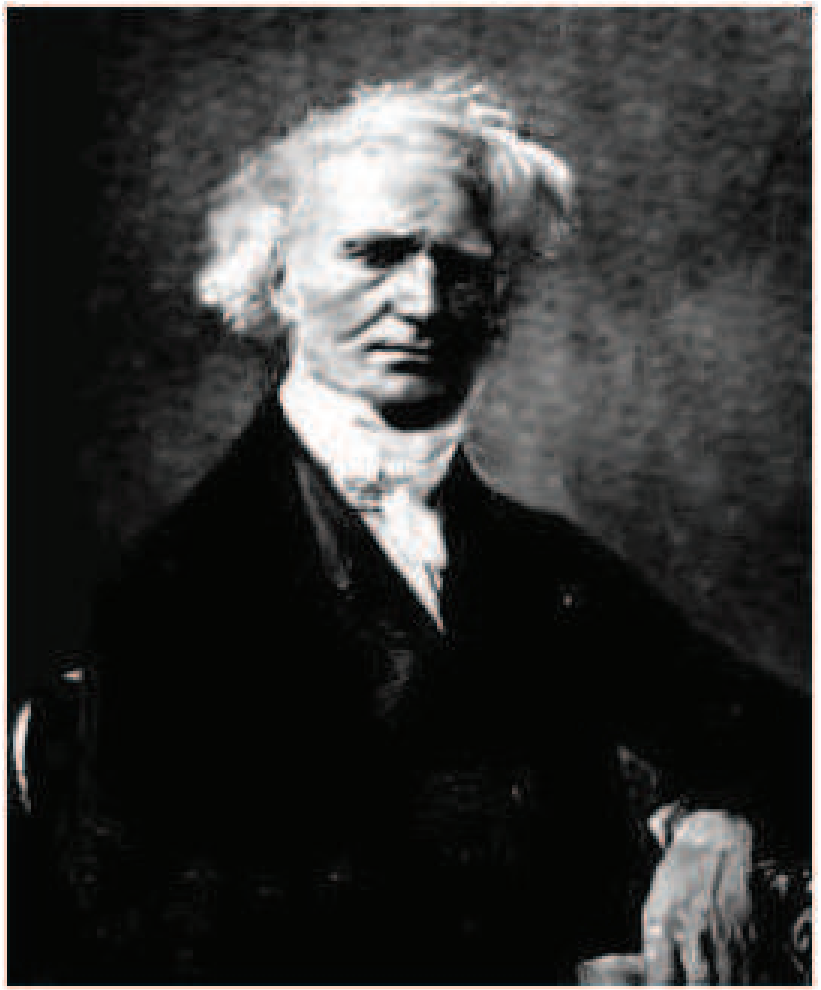
Phyllotaxis



55 spirals counterclockwise

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Abraham de Moivre
1667–1754



Jacques Philippe Marie Binet
1786–1856

Questions

1. What is the parity (evenness/oddness) of F_n?
2. Use the recurrence relation to define F_n for negative integers n.
3. Find a formula for finite Fibonacci series: $F_1 + F_2 + \cdots + F_n$.
4. Find a formula for: $F_1 + F_3 + \cdots + F_{2n-1}$.
5. Find a formula for: $F_1^2 + F_2^2 + \cdots + F_n^2$.
6. Use the fact that $\phi^2 = \phi + 1$ to find a formula for ϕ^n.
7. Find a formula for F_n involving ϕ, but not $1/\phi$.
8. What is $\lim_{n\to\infty} \frac{F_{n+1}}{F_n}$?

Phyllotaxis

calla lily: 1 petal

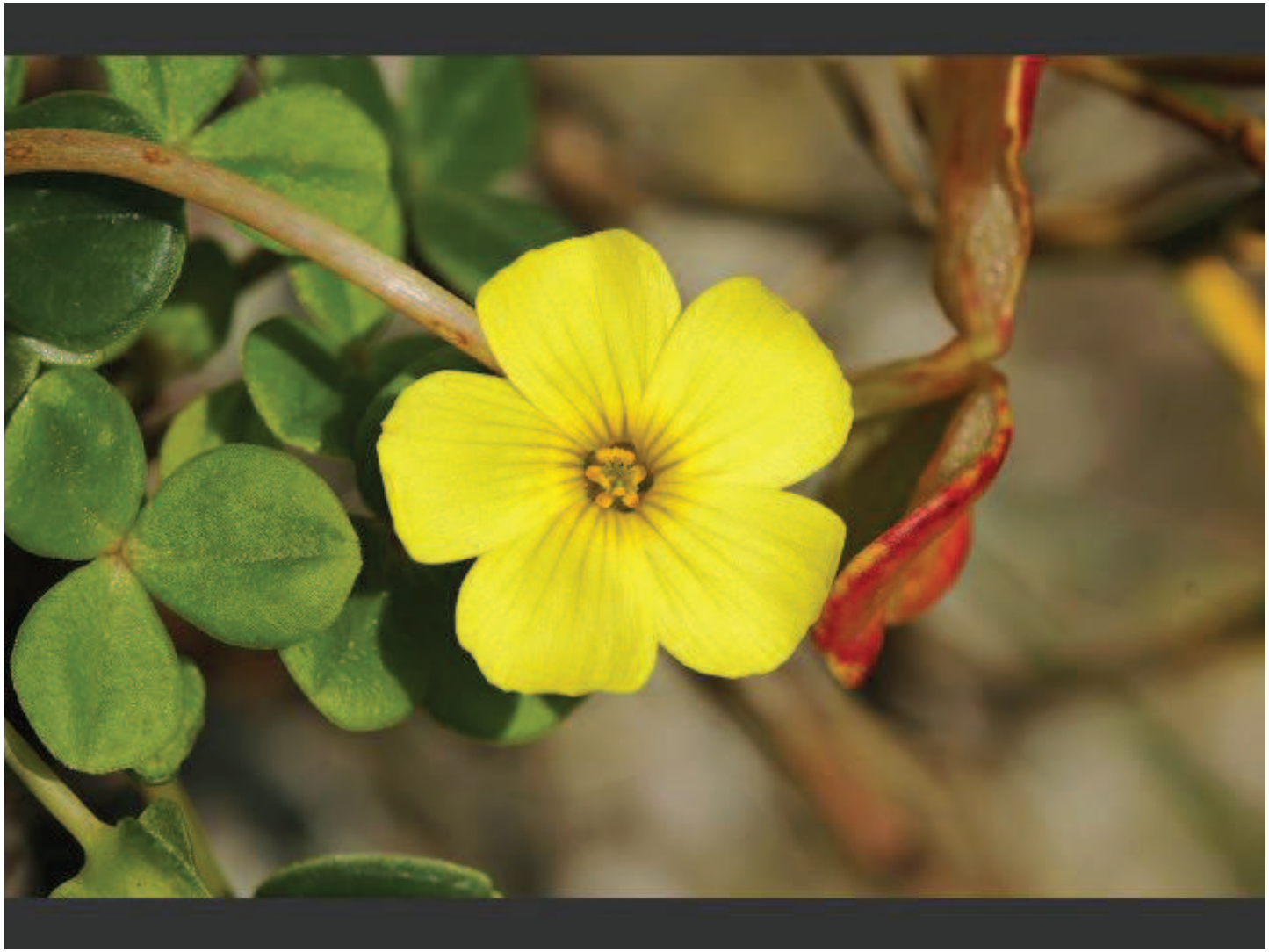
Phyllotaxis

euphorbia: 2 petals

Phyllotaxis

trillium: 3 petals

Phyllotaxis



buttercup: 5 petals

Phyllotaxis

columbine: 5 petals

Phyllotaxis

delphinium: 5 petals

Phyllotaxis

bloodroot: 8 petals

Phyllotaxis

delphinium: 8 petals

Phyllotaxis

signet marigold: 8 petals

Phyllotaxis

black-eyed Susan: 13 petals

Phyllotaxis

13 petals

Phyllotaxis

chrysanthemum carinatum: 21 petals

Phyllotaxis

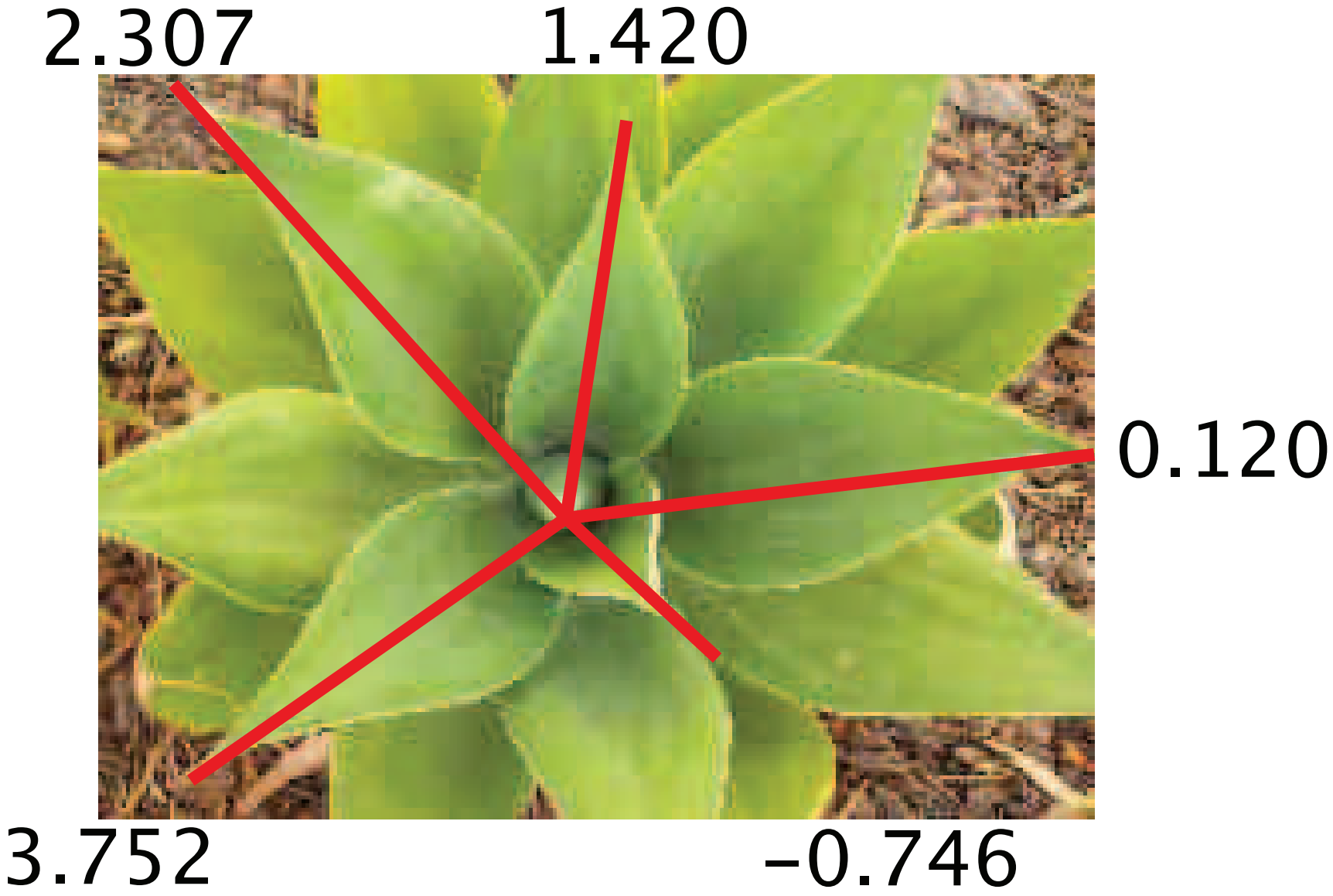
daisy: 21 petals

Phyllotaxis

sunflower: 21 petals

Phyllotaxis

Phyllotaxis



Questions

1. What are the first several convergents of $\psi = 1 + \sqrt{2}$?

2. Find a quadratic equation satisfied by ψ (and $-1/\psi$).

3. What is the corresponding recurrence relation?

4. What are the numbers analogous to the Fibonacci numbers?

And some geometry questions ...