D

Corrections and comments on Homework 7 (280A)

Problem 21 in Section 5.10 of Resnick should read,

$$\frac{d}{ds}P(s) = \sum_{k=1}^{\infty} k p_k s^{k-1} \text{ for } s \in [0,1].$$

Note that $P(s) = \sum_{k=0}^{\infty} p_k s^k$ is well defined and continuous (by DCT) for $s \in [-1,1]$. So the derivative makes sense to compute for $s \in (-1,1)$ with no qualifications. When $s = 1$ you should interpret the derivative as the one sided derivative

$$\frac{d}{ds}|_1 P(s) := \lim_{h \downarrow 0} \frac{P(1) - P(1-h)}{h}$$

and you will need to allow for this limit to be infinite in case $\sum_{k=1}^{\infty} k p_k = \infty$. In computing $\frac{d}{ds}|_1 P(s)$, you may wish to use the fact (draw a picture or give a calculus proof) that

$$\frac{1-s^k}{1-s} \uparrow k \text{ and } s \uparrow 1.$$

Hint for Exercise 8.20: Start by observing that

$$\begin{aligned}\mathbb{E}\left(\frac{S_n}{n} - \mu\right)^4 d\mu &= \mathbb{E}\left(\frac{1}{n}\sum_{k=1}^{n}(X_k - \mu)\right)^4 \\ &= \frac{1}{n^4}\sum_{k,j,l,p=1}^{n} \mathbb{E}\left[(X_k - \mu)(X_j - \mu)(X_l - \mu)(X_p - \mu)\right].\end{aligned}$$

Then analyze for which groups of indices (k,j,l,p);

$$\mathbb{E}\left[(X_k - \mu)(X_j - \mu)(X_l - \mu)(X_p - \mu)\right] \neq 0$$

by making use of the following result proved in class.

Proposition D.1. *Suppose that $(\Omega, \mathcal{B}, P)$ is a probability space and $\{Z_j\}_{j=1}^n$ are independent integrable random variables. Then $\prod_{j=1}^n Z_j$ is also integrable and*

$$\mathbb{E}\left[\prod_{j=1}^{n} Z_j\right] = \prod_{j=1}^{n} \mathbb{E}Z_j.$$