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# Construction of diffusions on path and loop spaces of compact Riemannian manifolds

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***Abstract*** – *In this Note we show how to construct an "Ornstein-Uhlenbeck" like process on the based path and pinned loop space of a compact Riemannian manifold without boundary. This is achieved by first constructing a natural Dirichlet form and then using the general theory of Dirichlet forms to construct a diffusion process.*

## Construction de diffusion sur les espaces de chemins ou de lacets d'une variété riemannienne compacte

***Résumé*** – Nous montrons dans cette Note comment construire un processus du type « Ornstein-Uhlenbeck » sur les chemins et sur les lacets avec points de base sur une variété riemannienne compacte. La méthode repose sur la construction d'une forme de Dirichlet naturelle et ensuite sur l'utilisation de la théorie générale des formes de Dirichlet qui permet de construire la diffusion cherchée.

***Version française abrégée*** – Considérons une variété riemannienne compacte $(M, g)$, de dimension $d$, sans bord. Supposons qu'une connection $\nabla$, compatible avec $g$, de torsion antisymétrique ait été choisie (par exemple la connection de Levi-Civita). On fixe un point de base $o \in M$. Soit $W(M) \subset C([0, 1], M)$ les chemins sur M, issus du point $o$. Soit $\nu$ la mesure de Wiener concentrée sur $W(M)$. Notons par $T_o M$ le plan tangent à M en $o$.

Pour chaque fonction $h : [0, 1] \to T_o M$, absolument continue et pour toute fonction $C^\infty$, cylindrique : $F(\sigma) = f(\sigma(s_1), \ldots, \sigma(s_n))$, on définit la $h$-dérivée $\partial_h F$ par l'équation (1). Dans l'équation (1) $H_s(\sigma)$ dénote le transport parallèle stochastique le long de $\sigma$, jusqu'au temps $s$. Définissons une forme quadratique symétrique, à domaine dense, $(\mathscr{E}, \mathscr{F}C^\infty)$ sur $L^2(\nu) := L^2(W(M); \nu)$ par l'équation (5) ou $\mathscr{F}C^\infty$ dénote les fonctions $C^\infty$ cylindriques. Dans l'équation (5), $\{h_n\}_{n=1}^{+\infty}$ est une base orthonormée de

$$H = \left\{ h : [0, 1] \to T_o M : h(0) = 0 \text{ et } (h, h)_H = \int_0^1 |h'(s)|^2 \, ds < \infty \right\}.$$

Utilisant la formule d'intégration par parties de [6], le calcul des dérivées des fonctions composées et utilisant [9], Proposition I.4.10 on obtient que la fermeture $(\mathscr{E}, D(\mathscr{E}))$ de $(\mathscr{E}, \mathscr{F}C^\infty)$ existe et est une forme de Dirichlet symétrique sur $L^2(\nu)$ (*cf.* Lemma 3).

Le théorème 4 contient le résultat principal de cette Note : la forme de Dirichlet $(\mathscr{E}, D(\mathscr{E}))$ est une forme locale, quasi-régulière sur $L^2(\nu)$. En appliquant la théorie générale (*cf.* [9]) il en résulte immédiatement qu'il existe un processus de diffusion à valeurs dans $W(M)$, associé à $(\mathscr{E}, D(\mathscr{E}))$. Ce processus est conservatif. La partie principale de la preuve consiste à montrer : 1. que la capacité associée à $(\mathscr{E}, D(\mathscr{E}))$ est essentiellement concentrée sur des compacts et 2. que la forme $(\mathscr{E}, D(\mathscr{E}))$ est locale. Ces deux faits sont établis dans la proposition 5.

Nous concluons en observant que les résultats ci-dessus restent valables pour la mesure de Wiener du pont brownien. Les seuls changements à faire sont :

(i) H doit être remplacé par $H_0 = \{h \in H; h(1) = 0\}$,

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(ii) utiliser la formule d'intégration par parties établie en [7] pour le pont brownien.

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This paper was motivated by the recent beautiful results in [1], [2]. It will be primarily written for the *path space case*. At the end we will explain the minor modifications needed to cover the *pinned loop space case*.

Let $(M, g, \nabla, o)$ be given, where M is a $d$-dimensional compact Riemannian manifold without boundary, $g$ is a Riemannian metric on M, $\nabla$ is a $g$-compatible covariant derivative, and $o \in M$ is a fixed base point. It will **always** be assumed that the covariant derivative $\nabla$ is *torsion skew symmetric* – *i.e.* if T is the torsion tensor of $\nabla$, then $g\langle T\langle X, Y\rangle, Y\rangle \equiv 0$ for all vector fields X and Y on M. We denote by $W(M)$ the set of continuous paths $\sigma:[0,1] \mapsto M$ such that $\sigma(0)=o$ and let $\mathscr{L}(M)=\{\sigma \in W(M) \mid \sigma(1)=o\}$ be the paths in $W(M)$ which also end at $o$. Both spaces we equip with the topology of uniform convergence. Wiener measure on $W(M)$ (the law of Brownian motion on M starting at $o \in M$) will be denoted by $\nu$, and the pinned Wiener measure concentrated on $\mathscr{L}(M)$ will be denoted by $\nu_o$. The corresponding real $L^2$-spaces are denoted by $L^2(\nu)$, $L^2(\nu_0)$ respectively. Recall that the coordinate maps $\Sigma_s: W(M) \to M$ given by $\Sigma_s(\sigma)=\sigma(s)$ are M-valued semi-martingales relative to both of the measures $\nu$ and $\nu_o$. Therefore it is possible to define a stochastic parallel translation operator $H_s(\sigma): T_o M \to T_{\sigma(s)} M$ for $\nu$ or $\nu_o$ almost every path $\sigma$.

In preparation for defining the *h-derivative* and the corresponding *gradient* we introduce the reproducing kernel Hilbert space H of functions $h:[0,1] \to T_o M$ such that $h(0)=0$, $h$ is absolutely continuous, and $(h, h)_H := \int_0^1 |h'(s)|^2 ds < \infty$, where $|v|^2 := g_o\langle v, v\rangle$ for $v \in T_o M$. A function $F: W(M) \mapsto \mathbf{R}$ is said to be a *smooth cylinder function* if F can be represented as $F(\sigma)=f(\sigma(s_1), \ldots, \sigma(s_n))$ where $f: M^n \mapsto \mathbf{R}$ is a smooth function and $0 \leqq s_1 \leqq s_2 \leqq \ldots \leqq s_n \leqq 1$. Let $\mathscr{F}C^\infty$ denote the set of all smooth cylinder functions. Note that $\mathscr{F}C^\infty$ is dense in $L^2(\nu)$. Given $h \in H$ and a smooth cylinder function $F(\sigma)$ as above the $h$-derivative of F is

$$\partial_h F(\sigma) := \sum_{i=1}^{n} g_{\sigma(s_i)}\langle \nabla_i f(\boldsymbol{\sigma}), H_{s_i}(\sigma) h(s_i)\rangle \tag{1}$$

where $\boldsymbol{\sigma} := (\sigma(s_1), \ldots, \sigma(s_n))$, $\nabla_i f(\boldsymbol{\sigma}) \in T_{\sigma(s_i)} M$ is the gradient of the function $f$ relative to the $i$'th variable while holding the remaining variables fixed. We now restate Theorem 9.1 and Corollary 9.1 from Driver [6].

THEOREM 1. – *The formula in* (1) *for* $\partial_h F(\sigma)$ *is well defined up to* $\nu$*-equivalence independent of the choices made in representing* F *as a smooth cylinder function and the chosen version for the process* $H_s(\sigma)$. *Furthermore, the adjoint* $\partial_h^*$ *of* $\partial_h$ [*relative to the* $L^2(\nu)$*-inner product*] *contains in its domain the class of smooth cylinder functions and hence is densely defined. The operator* $\partial_h^*$ *acting on the smooth cylinder functions has the form* $\partial_h^* = -\partial_h + z(h)$, *where* $z(h)$ *is a function which is in* $L^p(d\nu)$ *for all* $p$.

DEFINITION 2. – Let F be a smooth cylinder function. The *gradient* of F is the $\nu$ *a.e.* defined function $DF: W(M) \to H$ which satisfies $\partial_h F(\sigma) = (DF(\sigma), h)_H$ for all $h \in H$.

Let $G:[0,1]^2 \to \mathbf{R}$ be the Green's function for the operator $-(d^2/ds^2)$ with Dirichlet boundary conditions as $s=0$ and Neumann boundary conditions at $s=1$. [Explicitly $G(s, t) := \min(s, t)$.] It is well known and easy to check that G is a reproducing kernel for H – that is for all $h \in \mathrm{H}$ and $s \in [0, 1]$, $h(s) = \int_0^1 (\partial/\partial u)\, \mathrm{G}(s, u)\, h'(u)\, du$. Using this fact and Equation (1) it is easy to verify that DF is given explicitly by

$$\mathrm{DF}(\sigma)(s) = \sum_{i=1}^{n} \mathrm{G}(s, s_i)\, \mathrm{H}_{s_i}(\sigma)^{-1} \nabla_i f(\boldsymbol{\sigma}), \tag{2}$$

where $\mathrm{F}(\sigma) = f(\boldsymbol{\sigma})$ as above.

Given two smooth cylinder functions F and K on W(M) define a positive definite symmetric bilinear form on $\mathrm{L}^2(\nu)$ by

$$\mathscr{E}(\mathrm{F}, \mathrm{K}) := \int_{\mathrm{W(M)}} (\mathrm{DF}(\sigma), \mathrm{DK}(\sigma))_{\mathrm{H}}\, \nu(d\sigma), \qquad \mathrm{F}, \mathrm{K} \in \mathscr{F}\mathscr{C}^{\infty}. \tag{3}$$

Using the explicit expression (2) for DF and the fact that G is a reproducing kernel it is easy to show

$$\mathscr{E}(\mathrm{F}, \mathrm{F}) = \sum_{i, j=1}^{n} \int_{\mathrm{W(M)}} \mathrm{G}(s_i, s_j)\, g_0 \langle \mathrm{H}_{s_i}(\sigma)^{-1} \nabla_i f(\boldsymbol{\sigma}), \mathrm{H}_{s_j}(\sigma)^{-1} \nabla_j f(\boldsymbol{\sigma}) \rangle\, \nu(d\sigma), \tag{4}$$

where $\mathrm{F}(\sigma) = f(\boldsymbol{\sigma})$ as above.

LEMMA 3. – *The densely defined quadratic form* $(\mathscr{E}, \mathscr{F}\mathscr{C}^{\infty})$ *is closable on* $\mathrm{L}^2(\nu)$ *and its closure* $(\mathscr{E}, \mathrm{D}(\mathscr{E}))$ *is a symmetric Dirichlet form on* $\mathrm{L}^2(\nu)$ (*in the sense of* [8], [9]).

*Proof.* – Let $\{h_n\}_{n=1}^{\infty}$ be an orthonormal basis for H. Then

$$(\mathrm{DF}(\sigma), \mathrm{DF}(\sigma))_{\mathrm{H}} = \sum_{n=1}^{\infty} (\partial_{h_n} \mathrm{F}(\sigma))^2$$

and so

$$\mathscr{E}(\mathrm{F}, \mathrm{F}) = \sum_{n=1}^{\infty} \| \partial_{h_n} \mathrm{F} \|^2_{\mathrm{L}^2(d\nu)}. \tag{5}$$

The closability of $\mathscr{E}$ is now a routine consequence of Theorem 1, which implies that each of the operators $\partial_{h_n}$ is closable in $\mathrm{L}^2(\nu)$. The fact that $(\mathscr{E}, \mathrm{D}(\mathscr{E}))$ is a Dirichlet form is a direct consequence of the chain rule and [9], Proposition I, 4.10 (*see* also [8]). □

We now come to the main theorem.

THEOREM 4. – (i) $(\mathscr{E}, \mathrm{D}(\mathscr{E}))$ *is a* local quasi-regular *Dirichlet form on* $L^2(\nu)$.

(ii) *There exists a diffusion process* $\mathbf{M} = (\Omega, \mathscr{F}, (\mathscr{F}_t)_{t \geq 0}, (\mathrm{X}_t)_{t \geq 0}, (\mathrm{P}_z)_{z \in \mathrm{W(M)}})$ *associated with* $(\mathscr{E}, \mathrm{D}(\mathscr{E}))$, i.e. *for all* $u \in \mathrm{L}^2(\nu)$, *bounded, and* $t > 0$,

$$\int u(\mathrm{X}_t)\, d\mathrm{P}_z = \mathrm{T}_t u(z) \qquad \textit{for} \quad \nu\textit{-a.e.}\ z \in \mathrm{W(M)}$$

*where* $\mathrm{T}_t := e^{t\mathrm{L}}$, $t \geqq 0$, *and* L *is the* generator *of* $(\mathscr{E}, \mathrm{D}(\mathscr{E}))$.

Part (ii) follows from part (i) by [9], Theorem IV 3.5 and Theorem V 1.12 (*cf.* also [4]). To show (i), since $\mathscr{F}\mathscr{C}^{\infty}$ is dense in $\mathrm{D}(\mathscr{E})$ w.r.t. $\mathscr{E}_1^{1/2}$ [where $\mathscr{E}_1 := \mathscr{E} + (\ ,\ )_{\mathrm{L}^2(\nu)}$] and $\mathscr{F}\mathscr{C}^{\infty}$ separates the points of W(M), according to the definition of quasi-regularity (*cf.* [9], Definition IV 3.1], [4]) and by [9], Theorem III 2.11, we only have to prove:

PROPOSITION 5. – (i) *The capacity* Cap *associated with* $(\mathscr{E}, \mathrm{D}(\mathscr{E}))$ *is* tight, *i.e.* $\mathrm{Cap}(\mathrm{W(M)} \setminus \mathrm{K}_n) \xrightarrow[n \to \infty]{} 0$ *for some compact* $\mathrm{K}_n \subset \mathrm{W(M)}$, $n \in \mathbb{N}$.

(ii) $(\mathcal{E}, D(\mathcal{E}))$ *is* local, *i.e.* $\mathcal{E}(u, v)=0$ *if* $u, v \in D(\mathcal{E})$ *with* $\operatorname{supp}[|u| . \nu]$, $\operatorname{supp}[|v| . \nu]$ *compact and disjoint.*

Before we prove Proposition 5 we recall that for open $U \subset W(M)$

$$\operatorname{Cap}(U) := \inf \{ \mathcal{E}_1(F, F) \mid F \in D(\mathcal{E}),\ F \geqq 1 \text{ on } U\ \nu\text{-}a.e. \}$$

and arbitrary $A \subset W(M)$

$$\operatorname{Cap}(A) := \inf \{ \operatorname{Cap}(U) \mid A \subset U,\ U \subset E,\ U \text{ open} \}.$$

*Remark* 6. — (i) It is easy to see that in Theorem 4, $T_t 1 = 1$ for all $t \geqq 0$, hence **M** is *conservative.*

(ii) Part (ii) of Theorem 4 can also be derived directly from Proposition 5 by virtue of [5], Theorem 2.7.

(iii) For a proof of Proposition 5 (i) for a more general class of Dirichlet forms, namely those with a *square field operator*, we refer to [11].

Now we turn to the proof of Proposition 5. We shall prove part (i) by adapting a method due to B. Schmuland (*cf.* [10]). We denote the closure of D also by D. As in [10], Proof of Lemma 1.2 (or [9], Proof of Lemma IV 4.1) one proves using the chain rule for D that for all $F, K \in D(\mathcal{E})$

$$(6) \qquad (D(F \vee K), D(F \vee K))_H^{1/2} \leqq (DF, DF)_H^{1/2} \vee (DK, DK)_H^{1/2}$$

and

$$(7) \qquad (D(F \wedge K), D(F \wedge K))_H^{1/2} \leqq (DF, DF)_H^{1/2} \vee (DK, DK)_H^{1/2}$$

where $\vee$, $\wedge$ denotes sup, inf respectively. By the Whitney imbedding theorem we may assume without loss of generality that M is an imbedded submanifold of $\mathbf{R}^n$ for some integer $n$. Let $x^i$ for $i=1, 2, \ldots n$ denote the standard linear coordinates on $\mathbf{R}^n$, which may also be considered to be smooth functions on M. Let $\{s_k\}_{k=1}^{\infty}$ be a countable dense subset of $[0, 1]$. For $\sigma, \tau \in W(M)$ define $\rho(\sigma, \tau) := \sup |x^i(\sigma(s_k)) - x^i(\tau(s_k))|$, where the sup is taken over all $i=1, 2, \ldots, n$ and for all positive integers $k$. It is clear that $\rho$ is compatible with the topology of uniform convergence on $W(M)$.

For the moment fix $i$, $k$, and $\tau$ and consider the smooth cylinder function

$$F(\sigma) := x^i(\sigma(s_k)) - x^i(\tau(s_k)).$$

By equation (2) we see that

$$(8) \quad (DF(\sigma), DF(\sigma))_H = G(s_k, s_k)\, g_o \langle H_{s_k}(\sigma)^{-1} \nabla x^i(\sigma(s_k)),\ H_{s_k}(\sigma)^{-1} \nabla x^i(\sigma(s_k)) \rangle.$$

Using the facts that G is bounded by one and $H_{s_k}(\sigma)$ is an isometry, it follows from equation (8) that $(DF(\sigma), DF(\sigma))_H \leqq C$, where $C := \max \{ g \langle \nabla x^i(m), \nabla x^i(m) \rangle \mid m \in M$ and $i=1, 2, \ldots, n \}$. Because M is compact it follows that $C < \infty$. Now by exactly the same arguments as in [10], Proof of Proposition 3.1 it follows from (6) that

$$(9) \qquad \rho_\tau := \rho(\tau, .) \in D(\mathcal{E}) \quad \text{for all } \tau \in W(M)$$

and

$$(10) \qquad (D\rho_\tau, D\rho_\tau)_H \leqq C \quad \text{for all } \tau \in W(M).$$

Let $\{ \tau_k \mid k \in \mathbb{N} \}$ be dense in $W(M)$. Set

$$w_n := \inf_{1 \leqq k \leqq n} \rho_{\tau_k}, \qquad n \in \mathbb{N}.$$

Then $w_n \downarrow 0$ as $n \to \infty$ and by (7), (9) and (10) and the same arguments as in [10], Proof of 3.1 it follows that

$$w_n \to 0 \quad \text{quasi-uniformly on } W(M) \text{ as } n \to \infty,$$

*i.e.* there exists closed $F_m \subset W(M)$, $m \in \mathbb{N}$, such that $w_n \to 0$ uniformly on each $F_m$ as $n \to \infty$ and $\lim_{m \to \infty} \mathrm{Cap}(W(M) \setminus F_m) = 0$. By definition of $(w_n)_{n \in \mathbb{N}}$ it now follows that each $F_m$ it totally bounded, hence compact, and Proposition 5 (i) is proved.

To prove (ii) note that by a simple approximation argument

$$D(u.v) = u.D(v) + v\,D(u) \quad \text{for all bounded } u,\, v \in D(\mathscr{E}). \tag{11}$$

Then (ii) is an immediate consequence of the following fact which is now a special case of [9], Chap. VI, Proposition 1.7, *see* also Chap. VI, Example 1.13 (ii), since we already know that $(\mathscr{E}, D(\mathscr{E}))$ is quasi-regular.

PROPOSITION 7. — *For every open* $U \subset W(M)$ *there exists* $u \in D(\mathscr{E})$ *such that* $0 \leqq u \leqq 1_U$ (= *indicator function of* U) *and* $u > 0$ $\nu$-*a. e. on* U.

As stated at the beginning, the above results are also true for the pinned loop space setting. The required modifications are as follows. Let $H_o = \{ h \in H \mid h(1) = 0 \}$, and

$$G_0(s,t) := \begin{cases} s(1-t) & \text{if } s \leqq t \\ t(1-s) & \text{if } s \geqq t \end{cases}$$

which is the Green's function for the operator $-(d^2/ds^2)$ with Dirichlet boundary conditions at both $s=0$ and $s=1$. (Note: that $G_0$ is a reproducing kernel for $H_o$.) It is shown in [7] that Theorem 1 still holds if Wiener measure $\nu$ is replaced by pinned Wiener measure $\nu_o$ and the function $h$ is restricted to be in $H_o \cap C^1$. Definition 2 should be replaced by

DEFINITION 2′ — The *gradient* of F is now the $\nu_o$-*a. e.* defined function $D_o F : \mathscr{L}(M) \to H_o$ such that $\partial_h F(\sigma) = (D_o F(\sigma), h)_H$ for all $h \in H_o$.

Equation (2) still holds provided G is replaced by $G_0$. We now define $\mathscr{E}_0$ by the right member of (3) with (D, H, W(M), $\nu$) replaced by ($D_o$, $H_o$, $\mathscr{L}(M)$, $\nu_o$). The proof of Lemma 3 is still valid provided we choose an orthonormal basis $\{h_n\}$ for $H_o$ such that $\{h_n\} \subset H_o \cap C^1$. Thus we may define $(\mathscr{E}_0, D(\mathscr{E}_0))$ to be the form closure of $(\mathscr{E}_0, \mathscr{F}\mathscr{C}^\infty|_{\mathscr{L}(M)})$. Note that $\mathscr{F}\mathscr{C}^\infty|_{\mathscr{L}(M)}$ is dense in $L^2(\nu_0)$. The same proof given in Theorem 4 with (D, H, W(M), $\nu$) replaced by ($D_o$, $H_o$, $\mathscr{L}(M)$, $\nu_o$) proves:

THEOREM 4′ — (i) $(\mathscr{E}_0, D(\mathscr{E}_0))$ *is a local quasi-regular Dirichlet form on* $L^2(\nu_0)$.

(ii) *There exists a conservative diffusion process* **M** *with state space* $\mathscr{L}(M)$ *associated with* $(\mathscr{E}_0, D(\mathscr{E}_0))$.

For an analogue of Theorem 4′ for the free loop space over $\mathbb{R}^d$ including rotational invariance we refer to [3]. The results of this paper were announced on the *Third European Symposium on Analysis and probability*, Paris, January 3-10, 1992, where also part of this work was done. Both authors would like to express their gratitude to the organizers, in particular to P. Malliavin who also kindly added the French abridged version to this paper.

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