In[136]:=
```
(* Problem 1 *)
```

In[137]:=
```
(* on 1,-1 matrices  *)
```

```
(* The file containing the slides of my talk *)

     (* is the clickable item "Montecarlo talk" *)
```

In[138]:=
```
asu[v_] := Subscript[a, v]
```

In[139]:=
```
asu[{1, 2}]
```

Out[139]=
```
-1
```

In[140]:=
```
rollo := 1 - 2 Random[Integer, {0, 1}]
```

In[141]:=
```
Table [rollo, {40}]
```

Out[141]=
```
{1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1, -1, -1, -1, 1, 1, 1, -1, 1, 1,
  -1, -1, 1, -1, 1, -1, -1, 1, 1, -1, -1, -1, -1, 1, 1, -1, 1, -1, -1, -1}
```

In[142]:=
```
makemath[n_] := Table[Table[rollo, {n}], {n}]
```

In[143]:=
```
amath[n_] := Do[asu[{i, j}] = rollo, {i, 1, n}, {j, 1, n}]
```

In[144]:=
```
a[i_, j_] := asu[{i, j}]
```

In[145]:=
```
(*The next command constructs a matrix with entries  1,-1 *)
```

In[146]:=
```
amath[10]
```

In[147]:=
```
(*The next command prints out the resulting matrix *)
```

In[148]:=
```
MatrixForm[Table[a[i, j], {i, 1, 10}, {j, 1, 10}]]
```

Out[148]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \end{pmatrix}$$

In[149]:=
```
(* This procedure constructs a random vector Y and *)
(* computes the corresponding sum as described for Problem 1*)
```

In[150]:=
```
mksum[n_] := Block[{i, j, ut},
   (Y = Table[rollo, {n}];
    ut = 0;
    Do[te = Sum[a[i, j] Y[[j]], {j, 1, n}];
     If[te > 0, ut = ut + te, ut = ut - te], {i, 1, n}];
    Return[ut]
   )
  ]
```

In[151]:=
```
(* The next command prints out 300 samples of the sum*)
```

In[152]:=
```
boo = Table[mksum[10], {300}]
```

Out[152]=
```
{10, 22, 22, 10, 26, 22, 10, 14, 26, 18, 18, 26, 30, 30, 30, 30, 34, 18, 30, 18, 26, 18, 18, 30,
 26, 22, 34, 22, 22, 26, 10, 18, 26, 18, 30, 30, 26, 26, 26, 18, 26, 18, 14, 18, 22, 30, 22,
 22, 22, 14, 30, 30, 30, 22, 18, 38, 18, 26, 34, 26, 18, 34, 38, 14, 14, 26, 22, 18, 18, 22,
 30, 34, 26, 34, 34, 22, 22, 26, 22, 22, 22, 18, 26, 22, 26, 18, 30, 22, 22, 22, 18, 22, 30,
 34, 30, 26, 26, 14, 22, 26, 22, 26, 10, 30, 38, 34, 18, 30, 30, 14, 26, 26, 22, 22, 26, 26,
 42, 18, 30, 34, 26, 22, 34, 34, 26, 26, 6, 34, 34, 42, 26, 22, 18, 22, 22, 22, 42, 22, 42,
 30, 30, 6, 22, 18, 22, 30, 26, 30, 30, 22, 26, 34, 18, 22, 14, 18, 18, 26, 30, 26, 26, 22,
 18, 30, 26, 22, 26, 18, 26, 22, 22, 30, 14, 30, 22, 26, 22, 22, 22, 18, 26, 18, 26, 26, 38,
 30, 26, 30, 26, 18, 26, 26, 26, 26, 22, 18, 30, 26, 30, 34, 26, 38, 18, 10, 22, 38, 26, 10,
 30, 38, 14, 26, 30, 34, 34, 22, 22, 26, 18, 10, 26, 34, 30, 26, 18, 30, 30, 30, 22, 22, 42,
 26, 22, 26, 26, 38, 18, 18, 18, 30, 22, 30, 14, 22, 30, 6, 26, 22, 22, 18, 38, 26, 22, 38,
 34, 34, 14, 30, 30, 10, 34, 34, 26, 30, 30, 26, 34, 30, 30, 26, 26, 30, 30, 30, 22, 26, 22,
 26, 34, 34, 18, 34, 18, 18, 34, 14, 30, 30, 26, 14, 18, 18, 26, 22, 18, 30, 26, 38, 18, 26}
```

In[153]:=
```
adm[V_] := Sum[V[[i]], {i, 1, Length[V]}]
```

In[154]:=

In[155]:=
```
(* the next procedure computes the approximate expected  value for the sum*)
```

In[156]:=
```
                                (* Using the Central Limit Theorem*)
```

In[157]:=
```
exp[n_] := N[n Sqrt[n] Sqrt[2 / Pi]]
```

In[158]:=
```
exp[10]
```

Out[158]=
```
25.2313
```

In[159]:=
```
(* the next procedure computes the expected  value for the sum*)
```

In[160]:=
```
                                (* using the binomial distribution*)
```

In[161]:=
```
bino[n_] := N[Sum[n Binomial[n, i] Abs[2 i - n], {i, 0, n}] / 2^n]
```

In[162]:=
```
bino[10]
```

Out[162]=
```
24.6094
```

In[163]:=
```
(* counting the number of times the sum was above its expected value*)
```

In[164]:=
```
tst[u_] := If[u > bino[10], 1, 0]
```

In[165]:=
```
Map[tst, boo]
```

Out[165]=
```
{0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1,
 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 1, 1,
 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0,
 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1,
 1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 0,
 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0,
 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1,
 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1,
 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1}
```

In[166]:=
```
adm[%]
```

Out[166]=
```
168
```

In[167]:=
```
                                (* ALMOST HALF THE TIME !!*)
```

In[168]:=
```
Position[{1, 2, 3}, 2][[1]][[1]]
```

Out[168]= 2

In[169]:=
```
(* Problem 2 *)
```

In[170]:=
```
(* sum free subsets*)
```

In[171]:=
```
(* This procedure constructs a random set of n integers in the range  [1,M]  *)
```

In[172]:=
```
pickraset[M_, n_] := Block[{SS, ut},
   (SS = Sort[Table[Random[Integer, {1, M}], {n}]];
    Return[SS]
   )
  ]
```

In[173]:=
```
(* This procedure constructs all k-subsets of the set[1,n] *)
```

In[174]:=
```
mksubs[n_, k_] := Block[{sus, i},
   (sus = Subsets[Table[i, {i, 1, n}], {k}];
    Return[sus]
   )
  ]
```

In[175]:=
```
mksubs[5, 3]
```

Out[175]=
```
{{1, 2, 3}, {1, 2, 4}, {1, 2, 5}, {1, 3, 4},
 {1, 3, 5}, {1, 4, 5}, {2, 3, 4}, {2, 3, 5}, {2, 4, 5}, {3, 4, 5}}
```

In[176]:=
```
(* This procedure tests  if T is sum-free *)
```

In[177]:=
```
tstSumFree[T_] := Block[{n, ut, s},
  (n = Length[T];
   ut = True;
   Do[
    s = T[[i]] + T[[j]]; (* Print[i,j,"  -> ",s];*)
    If[MemberQ[T, s], (*Print[i,j];*)ut = False; Return[False]],
    {i, 1, n - 1}, {j, i + 1, n}];
   Return[ut])
 ]
```

In[178]:=
```
(* This procedure prints out all the triplets a,b,c in T with a+b=c *)
```

In[179]:=
```
PtstSumFree[T_] := Block[{n, ut, s},
  (n = Length[T];
   ut = True;
   Do[
    s = T[[i]] + T[[j]];
    If[MemberQ[T, s], Print[i, " , ", j, " -> ", Position[T, s][[1]][[1]]]; ut = False],
    {i, 1, n - 1}, {j, i + 1, n}];
   Return[ut])
 ]
```

In[180]:=
```
pcksubset[T_, SS_] := Table[SS[[T[[i]]]], {i, 1, Length[T]}]
```

In[181]:=
```
(* This procedure finds among all n/3-subsets of S the lex-first that is sum free *)
```

In[182]:=
```
gtsumfree[SS_] := Block[{n, ut, k, Tees, T, ST, i, j},
  (n = Length[SS];
   ut = {};
   k = Ceiling[n / 3];
   Tees = mksubs[n, k]; Print[n, " choose ", k, " = ", Length[Tees]];
   Do[T = Tees[[i]];
    ST = pcksubset[T, SS]; If[tstSumFree[ST], ut = ST;
     Print["subset ", i, "  is sumfree "]; Return[ut] ], {i, 1, Length[Tees]}];
   Return[ut]
  )
 ]
```

In[183]:=
```
subtra[x_, S_] := Table[S[[i]] - x, {i, 1, Length[S]}]
```

In[184]:=
```
SS = subtra[40, pickraset[150, 27]]
```

Out[184]=
```
{-29, -29, -29, -25, -24, -17, -16, -11, -4, 10, 20,
 26, 26, 29, 33, 43, 57, 59, 63, 72, 77, 80, 82, 83, 87, 101, 108}
```

In[185]:=
```
SSS = Union[SS]
```

Out[185]=
```
{-29, -25, -24, -17, -16, -11, -4, 10, 20,
 26, 29, 33, 43, 57, 59, 63, 72, 77, 80, 82, 83, 87, 101, 108}
```

In[186]:=
```
goo = {-37, -36, -35, -31, -28, -22, -19, -2,
  0, 3, 4, 6, 8, 11, 12, 15, 19, 24, 28, 33, 34, 50, 62, 77, 109}
```

Out[186]=
```
{-37, -36, -35, -31, -28, -22, -19, -2, 0,
 3, 4, 6, 8, 11, 12, 15, 19, 24, 28, 33, 34, 50, 62, 77, 109}
```

In[187]:=
```
Length[goo]
```

Out[187]=
```
25
```

In[188]:= **PtstSumFree[goo]**

```
1 , 9 -> 1

1 , 12 -> 4

1 , 16 -> 6

2 , 9 -> 2

2 , 13 -> 5

2 , 21 -> 8

3 , 8 -> 1

3 , 9 -> 3

3 , 11 -> 4

3 , 20 -> 8

3 , 22 -> 16

4 , 9 -> 4

4 , 10 -> 5

4 , 15 -> 7

4 , 21 -> 10

4 , 22 -> 17

5 , 9 -> 5

5 , 12 -> 6

5 , 19 -> 9

5 , 21 -> 12

5 , 23 -> 21

6 , 9 -> 6

6 , 10 -> 7

6 , 19 -> 12

6 , 20 -> 14

6 , 21 -> 15

6 , 22 -> 19

7 , 9 -> 7

7 , 17 -> 9

7 , 21 -> 16

8 , 9 -> 8

8 , 12 -> 11

8 , 13 -> 12

9 , 10 -> 10
```

```
9 , 11 -> 11

9 , 12 -> 12

9 , 13 -> 13

9 , 14 -> 14

9 , 15 -> 15

9 , 16 -> 16

9 , 17 -> 17

9 , 18 -> 18

9 , 19 -> 19

9 , 20 -> 20

9 , 21 -> 21

9 , 22 -> 22

9 , 23 -> 23

9 , 24 -> 24

9 , 25 -> 25

10 , 13 -> 14

10 , 15 -> 16

11 , 13 -> 15

11 , 14 -> 16

11 , 16 -> 17

11 , 18 -> 19

12 , 19 -> 21

13 , 14 -> 17

15 , 22 -> 23

16 , 17 -> 21

16 , 23 -> 24

19 , 21 -> 23
```

Out[188]= False

In[189]:= **gtsumfree[SSS]**

```
24 choose 8 = 735 471

subset 18  is sumfree
```

Out[189]= {-29, -25, -24, -17, -16, -11, 10, 20}

In[190]:= **good = {3, 7, 10, 12, 15, 23, 26, 34, 47,**
49, 55, 62, 74, 81, 87, 88, 91, 102, 104, 114, 123, 124, 126}

Out[190]= {3, 7, 10, 12, 15, 23, 26, 34, 47, 49, 55, 62, 74, 81, 87, 88, 91, 102, 104, 114, 123, 124, 126}

In[191]:= **Length[good]**

Out[191]= 23

In[192]:= **PtstSumFree[good]**

```
1 , 2 -> 3

1 , 4 -> 5

1 , 6 -> 7

1 , 16 -> 17

1 , 21 -> 23

2 , 11 -> 12

2 , 13 -> 14

2 , 14 -> 16

3 , 14 -> 17

3 , 19 -> 20

3 , 20 -> 22

4 , 12 -> 13

4 , 18 -> 20

4 , 20 -> 23

5 , 8 -> 10

5 , 9 -> 12

5 , 15 -> 18

6 , 7 -> 10

6 , 14 -> 19

6 , 17 -> 20

7 , 11 -> 14

7 , 12 -> 16

7 , 16 -> 20

8 , 9 -> 14

9 , 11 -> 18

10 , 11 -> 19

10 , 13 -> 21
```

Out[192]= False

In[193]:= **gtsumfree[good]**

```
23 choose 8 = 490 314

subset 19 125  is sumfree
```

Out[193]= {3, 7, 12, 23, 34, 47, 49, 55}

In[194]:= **SSS = Union[pickraset[150, 27]]**

Out[194]= {15, 27, 38, 49, 54, 61, 63, 64, 67, 70, 84, 88, 97, 100,
 101, 106, 108, 110, 115, 119, 121, 125, 130, 132, 135, 141, 143}

In[195]:= **Length[SSS]**

Out[195]= 27

In[196]:= **PtstSumFree[SSS]**

```
1 , 4 -> 8

1 , 14 -> 19

1 , 16 -> 21

1 , 18 -> 22

1 , 19 -> 23

2 , 6 -> 12

2 , 10 -> 13

2 , 12 -> 19

2 , 17 -> 25

3 , 7 -> 15

3 , 10 -> 17

3 , 13 -> 25

4 , 6 -> 18

4 , 10 -> 20

5 , 6 -> 19

5 , 9 -> 21

6 , 8 -> 22

7 , 9 -> 23
```

Out[196]= False

In[197]:=
```
False
```

Out[197]=
```
False
```

In[198]:=
```
(* Problem 3 *)
```

In[199]:=
```
(* two-colorings of the edges of  Kn *)
```

In[200]:=
```
(*  with no monochromatic a-clique *)
```

In[201]:=
```
(* the expected number of monochromatic a-cliques in a random 2-coloring*)
```

In[202]:=
```
totexp[n_, a_] := N[Binomial[n, a] 2 / 2^(a (a - 1) / 2)]
```

In[203]:=
```
(* finding the smallest a for which this expectation is <1 *)
```

In[204]:=
```
nPfindbesta[n_] := Block[{E, a},
  (Do[
    E = totexp[n, a];
    If[E < 1, ut = a; Return[ut]], {a, 1, n}];
   Return[ut]
  )
 ]
```

In[205]:=
```
(* prints all the expectations for 1≤ a <= n *)
```

In[206]:=
```
findbesta[n_] := Block[{E, a},
  (Do[
    E = totexp[n, a];
    If[E < 1, ut = a; Print[" for n=", n,
       "  and   a = ", a, " The expectation is ", E]; Return[ut]], {a, 1, n}];
   Return[ut]
  )
 ]
```

In[207]:=
```
(* table of smallest a and predicted a for 10,= n ≤ 30 *)
```

In[208]:=
```
MatrixForm[Transpose[Table[{n, nPfindbesta[n], Round[N[2 Log[2, n]] ]}, {n, 10, 30}]]]
```

Out[208]//MatrixForm=

10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
5	5	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	8	8	8
7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	9	9	10	10	10	10

```
(* table of probabilities for the smallest   a for 10<= n ≤ 20 *)
```

In[210]:=
```
Do[findbesta[n], {n, 10, 20}]
```

```
 for n=10  and   a = 5 The expectation is 0.492188

 for n=11  and   a = 5 The expectation is 0.902344

 for n=12  and   a = 6 The expectation is 0.0563965

 for n=13  and   a = 6 The expectation is 0.104736

 for n=14  and   a = 6 The expectation is 0.183289

 for n=15  and   a = 6 The expectation is 0.305481

 for n=16  and   a = 6 The expectation is 0.48877

 for n=17  and   a = 6 The expectation is 0.755371

 for n=18  and   a = 7 The expectation is 0.0303497

 for n=19  and   a = 7 The expectation is 0.0480537

 for n=20  and   a = 7 The expectation is 0.0739288
```

In[211]:=
```
(* the expectations for n=25 *)
```

In[212]:=
```
Table[{a, totexp[25, a]}, {a, 1, 7}]
```

Out[212]=
```
{{1, 50.}, {2, 300.}, {3, 575.}, {4, 395.313}, {5, 103.77}, {6, 10.8093}, {7, 0.458431}}
```

In[213]:=
```
(*counting monochromatic a-cliques  *)
```

In[214]:=
```
ZOrollo := Random[Integer, {0, 1}]
```

In[215]:=
```
Csu[v_] := Subscript[C, v]
```

In[216]:=
```
(* Constructs a random 2-coloring of the edges of Kn  *)
```

In[217]:=
```
colorK[n_] := Do[Csu[{i, j}] = ZOrollo, {i, 1, n - 1}, {j, i + 1, n}]
```

In[218]:=
```
colorK[20]
```

In[219]:=

In[220]:=
```
CC[i_, j_] := Csu[{i, j}]
```

In[221]:=
```
(* A random  2-coloring for  n=20  *)
```

In[222]:=
```
GG20 = MatrixForm[Table[CC[i, j], {i, 1, 19}, {j, i + 1, 20}]]
```

Out[222]//MatrixForm=

1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0	1	1
0	0	1	1	1	1	1	0	0	1	0	1	0	1	1	0	1	0	
1	0	1	1	0	0	1	1	1	1	0	0	0	0	1	0	0		
0	1	1	0	0	0	1	0	1	0	0	0	0	1	0	1			
1	1	1	1	0	0	1	1	1	1	1	0	1	0	0				
0	1	1	1	1	0	0	0	0	0	1	0	1	0					
0	1	0	1	1	1	0	1	0	0	0	1	0						
0	1	0	0	0	0	1	0	0	1	1	0							
0	1	0	1	0	0	1	1	0	0	0								
0	1	1	0	1	1	1	0	0	0									
1	0	0	0	0	1	1	0	1										
1	0	0	1	0	1	0	1											
0	1	0	0	1	0	1												
1	1	1	0	0	1													
0	0	0	0	1														
0	0	0	0															
1	0	0																
1	0																	
0																		

In[223]:=
```
Binomial[20, 2]
```

Out[223]= 190

```
(* the number of 2-colorings of K20 *)
```

In[225]:=
```
2^Binomial[20, 2]
```

Out[225]= 1 569 275 433 846 670 190 958 947 355 801 916 604 025 588 861 116 008 628 224

In[226]:=
```
(* procedure to depict a 2 colored clique with vertex set T *)
```

In[227]:=
```
depict[T_] := Print[T, " -> ",
  MatrixForm[Table[CC[T[[i]], T[[j]]], {i, 1, Length[T] - 1}, {j, i + 1, Length[T]}]]]
```

In[228]:=
```
(* procedure to test if a clique is monochromatic *)
```

In[229]:=
```
tstmono[T_] := Block[{a, S, ut, i, j},
   (a = Length[T];
    S = Sum[CC[T[[i]], T[[j]]], {i, 1, a - 1}, {j, i + 1, a}];
    ut = (S == a (a - 1) / 2) || (S == 0);
    Return[ut]
   )
  ]
```

In[230]:=
```
tstmono[{1, 2, 3, 4, 5}]
```

Out[230]=
```
False
```

In[231]:=
```
(* procedure to count the number of monochromatic a-cliques *)
```

In[232]:=
```
countmono[n_, a_] := Block[{Tees, T, i, ut},
  (Tees = mksubs[n, a]; Print[Length[Tees]];
   ut = 0;
   Do[
    T = Tees[[i]]; If[tstmono[T], ut = ut + 1; depict[T]], {i, 1, Length[Tees]}];
   Return[ut]
  )
 ]
```

In[276]:=
```
NPcountmono[n_, a_] := Block[{Tees, T, i, ut},
  (Tees = mksubs[n, a]; (* Print[Length[Tees]];*)
   ut = 0;
   Do[
    T = Tees[[i]]; If[tstmono[T], ut = ut + 1], {i, 1, Length[Tees]}];
   Return[ut]
  )
 ]
```

In[234]:=
```
Binomial[20, 7]
```

Out[234]=
```
77 520
```

In[235]:=
```
Binomial[20, 2]
```

Out[235]=
```
190
```

In[236]:=
```
(* Table of expected number of Monocromatic a-cliques for n=20 *)
```

In[237]:=
```
MatrixForm[Transpose[Table[{a, totexp[20, a]}, {a, 2, 8}]]]
```

Out[237]//MatrixForm=

2	3	4	5	6	7	8
190.	285.	151.406	30.2813	2.36572	0.0739288	0.00093855

In[238]:=
```
(* constructs a random 2-coloting of K20  *)
```

In[239]:=
```
colorK[20]
```

In[240]:=
```
(* exhibits  all the nonochromatic 6-cliques for this 2-coloring *)
```

In[491]:=
```
countmono[20, 6]
```

```
38 760
```

```
{2, 4, 6, 15, 18, 20} -> {{0, 0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}
```

Out[491]=
```
1
```

In[489]:=
```
colorK[20]
```

In[243]:=
```
MatrixForm[Table[CC[i, j], {i, 1, 19}, {j, i + 1, 20}]]
```

Out[243]//MatrixForm=

$$\begin{pmatrix}
\{0,1,1,1,1,0,1,0,1,0,1,1,0,0,0,1,1,1,0\}\\
\{0,0,1,0,1,0,1,1,1,0,0,0,1,0,0,1,0,0\}\\
\{0,1,0,0,0,0,0,0,1,1,1,1,1,1,0,0,1\}\\
\{0,0,0,1,1,1,1,0,1,1,1,0,1,1,1,0\}\\
\{1,1,1,1,0,1,0,0,0,1,1,1,1,0,1\}\\
\{1,1,0,1,0,1,0,1,1,0,1,1,0,0\}\\
\{0,1,1,0,0,0,1,0,0,0,0,0,0\}\\
\{0,1,0,1,0,1,0,1,0,0,0,0\}\\
\{0,0,0,0,0,0,0,1,0,1,0\}\\
\{0,1,1,1,0,0,0,1,0,1\}\\
\{1,1,1,0,1,1,0,0,0\}\\
\{0,0,1,0,1,0,1,1\}\\
\{0,1,0,0,0,1,0\}\\
\{0,1,1,0,0,1\}\\
\{1,0,0,1,1\}\\
\{0,0,1,0\}\\
\{1,1,0\}\\
\{0,1\}\\
\{1\}
\end{pmatrix}$$

In[244]:=
```
totexp[20, 6]
```

Out[244]= 2.36572

In[484]:=
```
totexp[15, 5]
```

Out[484]= 5.86523

```
colorK[20]
```

In[531]:=
```
countmono[20, 5]
```

15 504

$$\{1,3,9,17,18\} \to \begin{pmatrix}\{1,1,1,1\}\\ \{1,1,1\}\\ \{1,1\}\\ \{1\}\end{pmatrix}$$

$$\{1,3,9,17,20\} \to \begin{pmatrix}\{1,1,1,1\}\\ \{1,1,1\}\\ \{1,1\}\\ \{1\}\end{pmatrix}$$

$$\{1,3,11,17,18\} \to \begin{pmatrix}\{1,1,1,1\}\\ \{1,1,1\}\\ \{1,1\}\\ \{1\}\end{pmatrix}$$

$$\{1,3,17,18,19\} \to \begin{pmatrix}\{1,1,1,1\}\\ \{1,1,1\}\\ \{1,1\}\\ \{1\}\end{pmatrix}$$

$$\{1,3,17,19,20\} \to \begin{pmatrix}\{1,1,1,1\}\\ \{1,1,1\}\\ \{1,1\}\\ \{1\}\end{pmatrix}$$

```
{2, 4, 6, 11, 20} -> {{{0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}}

{2, 4, 6, 18, 20} -> {{{0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}}

{3, 8, 9, 17, 18} -> {{{1, 1, 1, 1}, {1, 1, 1}, {1, 1}, {1}}}

{3, 8, 17, 18, 19} -> {{{1, 1, 1, 1}, {1, 1, 1}, {1, 1}, {1}}}

{4, 6, 7, 11, 19} -> {{{0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}}

{4, 7, 9, 11, 19} -> {{{0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}}

{5, 6, 10, 11, 19} -> {{{0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}}

{6, 7, 10, 11, 19} -> {{{0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}}

{6, 10, 11, 14, 19} -> {{{0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}}
```

Out[531]= 14

In[247]:=
```
(* The Metropolis Algorithm *)
```

In[248]:=
```
pairs[n_] := Flatten[Table[{i, j}, {i, 1, n - 1}, {j, i + 1, n}], 1]
```

In[249]:=
```
pairs[5]
```

Out[249]= {{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 3}, {2, 4}, {2, 5}, {3, 4}, {3, 5}, {4, 5}}

In[250]:=
```
rapair[n_] := Block[{PRS, m, ut, k, i, j},
  (PRS = pairs[n];
   m = Length[PRS];
   k = Random[Integer, {1, m}];
   {i, j} = PRS[[k]];
   Return[{i, j}];
  )
 ]
```

In[251]:=
```
pairs[10]
```

Out[251]=
```
{{1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6}, {1, 7}, {1, 8}, {1, 9}, {1, 10}, {2, 3}, {2, 4}, {2, 5},
 {2, 6}, {2, 7}, {2, 8}, {2, 9}, {2, 10}, {3, 4}, {3, 5}, {3, 6}, {3, 7}, {3, 8}, {3, 9},
 {3, 10}, {4, 5}, {4, 6}, {4, 7}, {4, 8}, {4, 9}, {4, 10}, {5, 6}, {5, 7}, {5, 8}, {5, 9},
 {5, 10}, {6, 7}, {6, 8}, {6, 9}, {6, 10}, {7, 8}, {7, 9}, {7, 10}, {8, 9}, {8, 10}, {9, 10}}
```

In[252]:=
```
rapair[10]
```

Out[252]=
```
{1, 3}
```

In[253]:=
```
Table[rapair [15], {30}]
```

Out[253]=
```
{{9, 15}, {4, 15}, {1, 9}, {3, 10}, {14, 15}, {1, 5}, {11, 15}, {5, 10}, {7, 15}, {4, 5},
 {1, 2}, {9, 14}, {10, 12}, {8, 12}, {5, 11}, {6, 12}, {6, 8}, {5, 15}, {4, 15}, {9, 15},
 {7, 9}, {2, 12}, {10, 12}, {1, 6}, {7, 13}, {2, 9}, {2, 8}, {10, 14}, {6, 9}, {10, 15}}
```

In[254]:=
```
change[pr_] := Block[{i, j},
  ({i, j} = pr;
   CC[i, j] = 1 - CC[i, j];
  )
 ]
```

In[255]:=
```
change[{5, 7}]
```

In[256]:=
```
CC[5, 7]
```

Out[256]=
```
0
```

In[257]:=
```
Random[Real]
```

Out[257]=
```
0.0462801
```

In[258]:=
```
flip[p_] := If[Random[Real] < p, 1, 0]
```

In[259]:=
```
Table[flip[.925], {30}]
```

Out[259]=
```
{1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0}
```

In[260]:=
```
showit[n_] := MatrixForm[Table[CC[i, j], {i, 1, n - 1}, {j, i + 1, n}]]
```

In[546]:=
```
metrostep[n_, a_, olct_] := Block[{mat, nct, i, j, s, ut, p, u, v},
   ( mat = Table[CC[i, j], {i, 1, n - 1}, {j, i + 1, n}];
    {u, v} = rapair [n];
    CC[u, v] = 1 - CC[u, v];
    nct = NPcountmono[n, a]; (* Print[" metro ",olct," ",nct];*)
    If[nct == 0, Print[MatrixForm[mat]]; ut = 0; Return[ut]];
    If [nct > olct,
     p = olct / nct; s = flip[p];
     If[s == 0,  CC[u, v] = 1 - CC[u, v];
      ut = olct, ut = nct ], ut = nct ];
     Return[ut])
  ]
```

In[477]:=
```
olct
```

Out[477]=
```
olct
```

In[508]:=
```
totexp[14, 5]
```

Out[508]=
```
3.91016
```

In[524]:=
```
colorK[14]
```

In[526]:= **showit[14]**

Out[526]//MatrixForm=

$$\begin{pmatrix} \{1,1,1,0,1,0,0,1,0,1,0,0,1\} \\ \{0,0,0,0,1,1,1,1,0,1,0,1\} \\ \{0,1,1,0,1,1,0,1,0,0,0\} \\ \{1,0,0,1,0,1,0,1,0,1\} \\ \{0,1,0,1,0,0,0,1,1\} \\ \{0,1,1,0,0,1,1,0\} \\ \{1,0,0,0,0,0,1\} \\ \{1,1,0,1,0,0\} \\ \{1,0,0,1,1\} \\ \{0,0,0,0\} \\ \{0,1,0\} \\ \{1,0\} \\ \{0\} \end{pmatrix}$$

In[527]:= **countmono[14, 5]**

2002

Out[527]= 0

In[540]:= **colorK[14]**

In[550]:= **countmono[14, 5]**

2002

$$\{1,2,4,9,14\} \to \begin{pmatrix} \{0,0,0,0\} \\ \{0,0,0\} \\ \{0,0\} \\ \{0\} \end{pmatrix}$$

$$\{1,5,8,11,14\} \to \begin{pmatrix} \{0,0,0,0\} \\ \{0,0,0\} \\ \{0,0\} \\ \{0\} \end{pmatrix}$$

$$\{1,6,7,10,13\} \to \begin{pmatrix} \{1,1,1,1\} \\ \{1,1,1\} \\ \{1,1\} \\ \{1\} \end{pmatrix}$$

$$\{2,3,5,10,14\} \to \begin{pmatrix} \{0,0,0,0\} \\ \{0,0,0\} \\ \{0,0\} \\ \{0\} \end{pmatrix}$$

$$\{5,6,8,11,14\} \to \begin{pmatrix} \{0,0,0,0\} \\ \{0,0,0\} \\ \{0,0\} \\ \{0\} \end{pmatrix}$$

$$\{5,8,10,11,14\} \to \begin{pmatrix} \{0,0,0,0\} \\ \{0,0,0\} \\ \{0,0\} \\ \{0\} \end{pmatrix}$$

Out[550]= 6

In[551]:= **m = 6**

Out[551]= 6

In[556]:= **ct = 0**

Out[556]= 0

In[557]:= **While[m > 0 && ct < 60, m = metrostep[14, 5, m]; ct = ct + 1; Print[ct, " ", m]]**

```
1  2
2  3
3  3
4  1
5  1
6  1
7  1
8  1
9  1
10  1
11  1
12  1
13  1
14  1
15  1
16  1
17  1
18  1
19  2
20  2
21  2
22  2
23  2
24  2
25  1
26  1
27  1
```

```
{0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0}
 {0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1}
  {1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0}
   {1, 1, 1, 0, 0, 1, 0, 0, 0, 0}
    {0, 1, 1, 0, 0, 1, 1, 1, 0}
     {1, 1, 0, 0, 0, 0, 1, 0}
      {0, 0, 0, 0, 1, 1, 1}
       {0, 1, 0, 0, 1, 0}
        {0, 1, 0, 1, 1}
         {1, 0, 1, 1}
          {1, 1, 1}
           {0, 0}
            {1}
```

```
28  0
```

In[394]:=
```
totexp[15, 4]
```

Out[394]=
```
42.6563
```

```
(* searching for a 2-coloring of the edges of Kn with some
  monochromatic a-cliques*)
```

In[461]:=
```
searchit[n_, a_, m_] := Block[{ci, monos},
  (ct = 0;
   monos = 0;
   While[ct < m
     && monos == 0,
    ct = ct + 1;
    colorK[n];
    monos = countmono[n, a];
    If[monos > 0, Print[ct, "  -> ", monos], Print[ct]]];
   Print["===="])
  ]
```

In[450]:=
```
totexp[15, 6]
```

Out[450]=
```
0.305481
```

In[462]:=
```
searchit[15, 5, 200]
```

```
3003
```

$$\{2, 6, 9, 10, 12\} \to \begin{pmatrix} \{1, 1, 1, 1\} \\ \{1, 1, 1\} \\ \{1, 1\} \\ \{1\} \end{pmatrix}$$

$$\{2, 9, 10, 12, 13\} \to \begin{pmatrix} \{1, 1, 1, 1\} \\ \{1, 1, 1\} \\ \{1, 1\} \\ \{1\} \end{pmatrix}$$

$$\{3, 4, 5, 6, 11\} \to \begin{pmatrix} \{0, 0, 0, 0\} \\ \{0, 0, 0\} \\ \{0, 0\} \\ \{0\} \end{pmatrix}$$

$$\{3, 4, 5, 10, 11\} \to \begin{pmatrix} \{0, 0, 0, 0\} \\ \{0, 0, 0\} \\ \{0, 0\} \\ \{0\} \end{pmatrix}$$

$$\{3, 4, 7, 10, 11\} \to \begin{pmatrix} \{0, 0, 0, 0\} \\ \{0, 0, 0\} \\ \{0, 0\} \\ \{0\} \end{pmatrix}$$

```
1  -> 5

====
```

In[472]:=
```
countmono[15, 5]
```

```
3003
```

$$\{3, 6, 9, 10, 12\} \to \begin{pmatrix} \{1, 1, 1, 1\} \\ \{1, 1, 1\} \\ \{1, 1\} \\ \{1\} \end{pmatrix}$$

Out[472]= 1

In[460]:=
```
totexp[16, 5]
```

Out[460]= 8.53125

In[469]:=
```
a = 2
```

Out[469]= 2

In[470]:=
```
ct = 0
```

Out[470]= 0

In[471]:=
```
While[a > 0 && ct < 30, a = metrostep[16, 5, a]; ct = ct + 1; Print[" -> ", ct, "   ", a]]
```

```
2 2
 -> 1   7
7 4
 -> 2   7
```

```
7 4
 -> 3  7
7 6
 -> 4  7
7 8
 -> 5  8
8 7
 -> 6  7
7 7
 -> 7  7
7 6
 -> 8  7
7 6
 -> 9  7
7 6
 -> 10  7
7 6
 -> 11  7
7 6
 -> 12  7
7 5
 -> 13  7
7 5
 -> 14  7
7 5
 -> 15  7
7 5
 -> 16  7
7 6
 -> 17  7
7 5
 -> 18  7
7 5
 -> 19  7
7 5
```

```
 -> 20  7

7 5

 -> 21  7

7 5

 -> 22  7

7 1

 -> 23  7

7 1

 -> 24  7

7 2

 -> 25  7

7 1

 -> 26  7

7 1

 -> 27  7

7 1

 -> 28  7

7 1

 -> 29  7

7 1

 -> 30  7
```

In[467]:= **countmono[15, 5]**

```
3003

{1, 3, 6, 8, 13} -> {{0, 0, 0, 0}, {0, 0, 0}, {0, 0}, {0}}

{2, 6, 9, 10, 12} -> {{1, 1, 1, 1}, {1, 1, 1}, {1, 1}, {1}}
```

Out[467]= 2

In[333]:= **a**

Out[333]= 0

In[436]:=
```
showit[16]
```

Out[436]//MatrixForm=

$$\begin{pmatrix}
\{0,1,1,1,0,1,0,0,1,1,1,0,1,0,1\} \\
\{0,1,0,1,1,1,1,1,0,0,1,1,0,1\} \\
\{0,0,1,0,1,1,0,0,1,1,1,0,1\} \\
\{0,0,1,1,0,0,1,1,0,1,0,0\} \\
\{0,1,1,1,0,0,0,1,0,1,0\} \\
\{1,0,0,0,0,1,1,1,0,1\} \\
\{0,1,0,0,1,0,0,0,0\} \\
\{1,0,0,1,0,1,0,0\} \\
\{1,0,1,1,1,0,0\} \\
\{0,0,1,0,1,1\} \\
\{1,1,0,1,1\} \\
\{1,0,1,1\} \\
\{0,0,1\} \\
\{0,0\} \\
\{0\}
\end{pmatrix}$$