Tuesday, February 24th 2015
MATH 15A

Practice Midterm II

Follow these instructions carefully.

1. No calculators or other electronic computational aids may be used during the exam.
2. You may have one page of notes, but no books or other assistance.
3. Write your name, PID, and section on the cover of your bluebook.
4. Show all your work in the bluebook.
5. No credit will be given for unsupported answers.
6. Present your answer clearly.
    a) Carefully indicate the number and letter of each question and question part.
    b) Try to present your answers in the same order they appear in the exam.
    c) Start each question on a new side of a page.

There are four questions. Each question is worth 10 points.

Question 1.
Let $\mathcal{L}$ be the set of lists of natural numbers. For each part below, give an example of a recursively defined function f with domain $\mathcal{L}$ that satisfies the condition named in each part. Note that you can use a different function for each part.

(a) f is a bijection

(b) f is not surjective

(c) f is not injective

Question 2.
Recall that the factorial function $n!$ is defined recursively by $0! = 1$ and for $n > 0$, $n! = (n-1)! \cdot n$.

Prove by induction that $n! \geq 2^n$ for $n \geq 4$. Be sure to indicate what kind of induction proof you are using.

Question 3.
Let $B(k)$ be a sequence defined by the following recurrence relation: $B(1) = 1$ and for $k \geq 2$, $B(k) = B(k-1) + k^2$. Let $h(k) = k(k+1)(2k+1)/6$.

Prove that for all $k \geq 1$ that $B(k) = h(k)$.

Question 4.
Prove the following claim by induction on n: If X is a finite set, with $|X| = n$, then the number of two element subsets of X is $n \cdot (n-1)/2$.

Hint: Take any $x \in X$, and form the set $X' = X \setminus \{x\}$. Since $|X'| = n-1$, the induction hypothesis applies to X'.