Name: ______________________________ **PID:** ________________

TA: _______________________ **Sec. No:** _____ **Sec. Time:** _____

Math 20C.
Final Examination
June 12, 2007

Turn off and put away your cell phone.
You may use any type of handheld calculator; no other devices are allowed on this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

1. (a) (3 points) Find a vector equation for the line through the point $(5, 1, 4)$ and perpendicular to the plane $x - 2y + z = 1$.

   (b) (3 points) At what point does the line intersect the plane $x - 2y + z = 1$?

#	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	6	
8	6	
9	6	
10	6	
Σ	60	

2. (6 points) Vector equations for two intersecting lines are given below:

$$\mathbf{r}_1(s) = \langle -1, 1, 5\rangle + s\langle -2, 3, 2\rangle$$
$$\mathbf{r}_2(t) = \langle 2, -4, 1\rangle + t\langle 1, -2, -2\rangle$$

(a) Find the point of intersection of the two lines.

(b) Find an equation for the plane containing both lines.

3. (6 points) A particle moves with position function

$$\mathbf{r}(t) = \langle 3t\,\ln(t),\ t,\ e^{-3t} \rangle.$$

Find the velocity, speed, and acceleration of the particle.

4. (6 points) Let $f(x, y) = y + x\cos(x - y)$.

(a) Find an equation for the plane tangent to $z = f(x, y)$ at the point $(2, 2, 1)$.

(b) Use a linear approximation to estimate the value of $f(2.1, 1.9)$.

5. (6 points) The electrical potential V in some region of space is

$$V(x, y, z) = x^3 + 2x^2y + 3xyz.$$

(a) Find the rate of change of the potential at the point $(-1, 1, 2)$ in the direction of the vector $v = \langle -2, 1, 2 \rangle$.

(b) In which direction does V change most rapidly at the point $(-1, 1, 2)$?

(c) What is the maximum rate of change of V at the point $(-1, 1, 2)$?

6. (6 points) Let $f(x, y) = x^3 + y^3 + 12xy$.

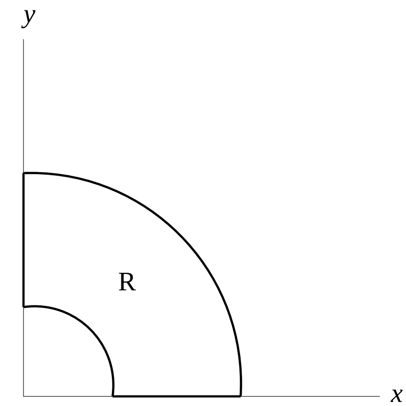
(a) Find the critical points of f.

(b) Use the second derivative test to classify each critical point of f as a local minimum, local maximum or saddle point.

7. (6 points) Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x - 2y + 3z$ subject to the constraint $x^2 + y^2 + z^2 = 42$.

8. (6 points) Compute $\displaystyle\int_{x=0}^{1}\int_{y=x}^{1} \sin(y^2)\, dy\, dx$ by first changing the order of integration.

9. (6 points) Let R be the region in the plane between the curves $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ with $x \geq 0$ and $y \geq 0$.



Compute

$$\iint_R \frac{y}{x^2 + y^2}\, dA.$$

(Hint: what would be the best coordinate system to use?)

10. (6 points) Let T be the solid tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$. Compute

$$\iiint_T xy\,dV.$$