Math 142B
Midterm Exam 1
August 11, 2011
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Instructions

1. **You may use any type of calculator, but no other electronic devices during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new page.**
5. **Read each question carefully, and answer each question completely.**
6. **Show all of your work; no credit will be given for unsupported answers.**

1. Let $f : [0,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & \text{if } x = \frac{1}{n} \text{ for some } n \in \mathbb{N}, \\ 0 & \text{otherwise.} \end{cases}$$

   Show the f is integrable on $[0,1]$ and determine the value of $\int_0^1 f$.

2. Let $f : [a,b] \to \mathbb{R}$ be a continuous function such that $\int_c^d f \geq 0$ for all c, d with $a \leq c < d \leq b$. Prove that $f(x) \geq 0$ for all $x \in [a,b]$.

3. Exhibit an example of a function $f : [0,1] \to \mathbb{R}$ that is unbounded.

4. For numbers $a_1, \ldots, a_n$, define $p(x) = a_1 x + a_2 x^2 + \cdots + a_n x^n$ for all x. Suppose that

$$\frac{a_1}{2} + \frac{a_2}{3} + \cdots + \frac{a_n}{n+1} = 0.$$

   Prove that there is an $x_0 \in (0,1)$ such that $p(x_0) = 0$.

5. Let $f : [a,b] \to \mathbb{R}$ be monotonically increasing.
   (a) Show that f is bounded on $[a,b]$.
   (b) Let P_n be a regular partition of $[a,b]$ into n partition intervals. Show that

$$U(f, P_n) - L(f, P_n) = \frac{[f(b) - f(a)][b - a]}{n}$$