Groups32 - Sample Session

This is the script of a sample session with Groups32. You should go through it while running Groups32 in a separate window. Try the commands even if you have not yet learned about them.

This program is based on the stored multiplication tables for the groups of order 1 to 32. A variety of commands can be applied to these groups. You can get a list of these commands at any time by typing “H”.

It is only necessary to type enough letters to distinguish a command from others – Groups32 will complete typing the name of the command for you and ask you for any additional information required.

In this sample session, what the user types is shown in red, and the remaining letters (supplied by the computer) are in blue. Everything in black is computer output.

Click on the blue letters to obtain more information about a command.

```
Forth for Windows 95, and NT
Compiled: July 26th, 1996, 12:05am
 Version: 3.2  Build: 0819  Release Build
Platform: Win32s on Windows Version: 3.10  Build: 167
64k bytes free
 3,008 Words in Application dictionary
 3,198 Words in System dictionary
 6,206 Words total in dictionaries

Platform: Win32Forth  ver 3.2
          An implementation of Forth for Windows
          Written by Tom Zimmer and Andrew McKewan
          For more information and most recent release
          see www.forth.org

Groups32
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                    Dept of Math - UCSD
Ver 6.3  - August 19, 1999

  CENTER       CENTRALIZER  CHART        CONJ-CLS
  COSETS       EVALUATE     EXAMPLES     GENERATE
  GROUP        HELP         INFO         ISOMORPHISM
  LEFT         NORMALIZER   ORDERS       PERMGRPS
  POWERS       QUIT         RESULT       RIGHT
  SEARCH       STOP         SUBGROUPS    TABLE
  X
```

Here is a listing, by group order, of all the numbered groups in the system.

Notice that there are two groups of order 4, but only one group each of orders 2,3,5,7. Does this suggest any conjectures?

```
G1>> CHART   Order of Groups (1-32 or 0) Number 0
Order               Groups of that order
  1   1               * = non-abelian
  2   2
  3   3
  4   4   5
  5   6
  6   7   8*
  7   9
  8  10  11  12  13* 14*
  9  15  16
 10  17  18*
 11  19
 12  20  21  22* 23* 24*
 13  25
 14  26  27*
 15  28
 16  29  30  31  32  33  34* 35* 36* 37* 38* 39* 40* 41* 42*
 17  43
 18  44  45  46* 47* 48*
 19  49
 20  50  51  52* 53* 54*
 21  55  56*
 22  57  58*
 23  59
 24  60  61  62  63* 64* 65* 66* 67* 68* 69* 70* 71* 72* 73*
     74*
 25  75  76
 26  77  78*
 27  79  80  81  82* 83*
 28  84  85  86* 87*
 29  88
 30  89  90* 91* 92*
 31  93
 32  94  95* 96  97* 98* 99* 100* 101* 102* 103* 104* 105* 106*
     107* 108* 109  110* 111* 112* 113* 114  115* 116* 117* 118*
     119* 120* 121* 122* 123* 124* 125* 126* 127* 128* 129  130*
     131* 132* 133* 134* 135* 136* 137* 138  139* 140* 141* 142*
     143* 144
```

Here we find the groups of specific order (12 and 6):

```
G1>> CHART   Order of Groups (1-32 or 0)
Number 12
      20   21   22*  23*  24*
        There are 5 Groups of order 12
        2 abelian and 3 non-abelian

G1>> CHART   Order of Groups (1-32 or 0)
Number 6
       7    8*
        There are 2 Groups of order 6
        1 abelian and 1 non-abelian
```

Here are multiplication tables for the two groups of order 6. The fact that #7 is commutative can be seen from the table. Notice that A is the identity element in all of the tables. Notice also that the Gx>> prompt indicates which group is the current group.

```
G1>> TABLE   for Group Number 7

  _A_B_C_D_E_F_
A |A B C D E F
B |B C D E F A
C |C D E F A B
D |D E F A B C
E |E F A B C D
F |F A B C D E

G7>> TABLE   for Group Number 8

  _A_B_C_D_E_F_
A |A B C D E F
B |B C A F D E
C |C A B E F D
D |D E F A B C
E |E F D C A B
F |F D E B C A
```

G8>> TABLE   for Group Number 8

```
   _A_B_C_D_E_F_
A  |A B C D E F
B  |B C A F D E
C  |C A B E F D
D  |D E F A B C
E  |E F D C A B
F  |F D E B C A
```

We read the multiplication tables so that a product x*y is the entry in the row of x and the column of y. In this table, BD=F but DB=E.
This is not a commutative group.
Notice also that A is the identity element.

The EVALUATE command computes a product of group elements and their inverses. Notice that elements can be entered either in upper or lower case -- so "bd" is the same as "BD".

```
G8>> EVALUATE   (use ' for inverse)
bd= F
G8>> EVALUATE   (use ' for inverse)
db= E
```

```
G8>> GENERATE   a subgroup using
set: { b }
{ A B C }
G8>> GENERATE   a subgroup using
set: { d }
{ A D }
G8>> GENERATE   a subgroup using
set: { bd }
{ A B C D E F }
```

Still in group 8, b generates a subgroup of order 3, d generates a subgroup of order 2, and together they generate the whole group. This also can be seen from the listing of all subgroups.

```
G8>> SUBGROUPS   of Group Number 8

   * = Normal subgroup
   Generators            Subgroup
  0  { }                 *{ A }
  1  { D }                { A D }
  2  { E }                { A E }
  3  { F }                { A F }
  4  { B }               *{ A B C }
  5  { B D }             *{ A B C D E F }
```

Here is a list of the elements of groups 7 and 8 by order. A is the identity, so it has order 1.

Notice that group 7 has two elements of order 6 (the same as the order of the group). What does this say about group 7?

```
G8>> ORDERS   for Group Number 7

Group number 7 of Order 6
    1 elements of order  1:   A
    1 elements of order  2:   D
    2 elements of order  3:   C E
    2 elements of order  6:   B F

G7>> ORDERS   for Group Number 8

Group number 8 of Order 6
    1 elements of order  1:   A
    3 elements of order  2:   D E F
    2 elements of order  3:   B C
    0 elements of order  6:
```

The list of commands can be printed at any time by typing HELP -- so you do not need to memorize them.

```
G8>> HELP

  CENTER        CENTRALIZER   CHART         CONJ-CLS
  COSETS        EVALUATE      EXAMPLES      GENERATE
  GROUP         HELP          INFO          ISOMORPHISM
  LEFT          NORMALIZER    ORDERS        PERMGRPS
  POWERS        QUIT          RESULT        RIGHT
  SEARCH        STOP          SUBGROUPS     TABLE
  X
```

```
G8>> COSETS   of subg generated by set: { d }

 Left Cosets                 Right Cosets
{ A D }                    { A D }
{ B F }                    { B E }
{ C E }                    { C F }

The subgroup { A D }  is NOT a NORMAL subgroup
```

The subgroup (of group 8) generated by d is not normal. One way to see this is to notice that the left cosets are not the same as the right cosets.

You can also compute the normalizer of a subgroup.

```
G8>> NORMALIZER   of subgroup gen by set: { b }
{ A B C D E F }

G8>> NORMALIZER   of subgroup gen by set: { d }
{ A D }
```

The command PERMGRPS switches to a subpackage. Notice that both the menu of commands and the prompt change when you are in this subpackage.

```
G8>> PERMGRPS

  CREATE       ELEMENTS     HELP         INFO
  INSTALL      MAIN         MULTIPLY     QUIT
  X
```

We will look at the PERMGRPS subpackage.
First, however, notice that you can get information about any command while using Groups32 by using the X or INFO command. The next command you type will be described without executing it.

```
PERM>> X
     This will provide information about the next
     command you use. INFO and X do the same thing
     but X is quicker to use.
```

Use the X command

```
PERM>> CREATE
      This will determine the subgroup of Sn generated by
   a given set of permutations (given as a product of
   cycles). You must put in n (for Sn) and then the
   generators using numbers 1..n  for example
   (1 2)(3 4 5).  The program will only compute groups
   up to order 51.  If the resulting group has order 32
   or less, you can install the table as one of the groups
   1-5.
```

Then type CREATE to find out what this command does

Now we actually execute the CREATE command to make a permutation group which has a given set of generators.

```
PERM>> CREATE
Subgroup of Sn -- what is n?  Number 4
     Put in generators as product of cycles.
     End with a blank line
Generator (1 2 3)
Generator (1 2 4)
Generator
Group is of order 12
A  ()                        B  (2 3 4 )                 C  (2 4 3 )
D  (1 2 )(3 4 )              E  (1 2 3 )                 F  (1 2 4 )
G  (1 3 2 )                  H  (1 3 4 )                 I  (1 3 )(2 4 )
J  (1 4 2 )                  K  (1 4 3 )                 L  (1 4 )(2 3 )
```

Installing a group as group 1 replaces the table for group 1 (originally the trivial group) with the table for the new group. This allows us to go back to the main menu and apply all of the operations to the newly generated group. The group we have just created is seen to be a non-abelian group of order 12. We then see that it has the same distribution of elements (of each order) as group 23. We next find an explicit isomorphism between these groups.

For two groups to be isomorphic, is it always enough that they have the same number of elements of each order?

```
PERM>> INSTALL
    Install as table k (1..5)  Number 1

PERM>> MAIN

  CENTER        CENTRALIZER  CHART        CONJ-CLS
  COSETS        EVALUATE     EXAMPLES     GENERATE
  GROUP         HELP         INFO         ISOMORPHISM
  LEFT          NORMALIZER   ORDERS       PERMGRPS
  POWERS        QUIT         RESULT       RIGHT
  SEARCH        STOP         SUBGROUPS    TABLE
  X
```

```
G1>> ORDERS   for Group Number 1

Group number 1 of Order 12
    1 elements of order  1:   A
    3 elements of order  2:   D I L
    8 elements of order  3:   B C E F G H J K
    0 elements of order  4:
    0 elements of order  6:
    0 elements of order 12:

G1>> CENTER   of Group Number 1
{ A }

G1>> CHART   Order of Groups (1-32 or 0) Number 12
       1*  20   21   22*  23*  24*
        There are 6 Groups of order 12
        2 abelian and 4 non-abelian
```

This new group (now group 1) must be essentially the same as one of the 5 groups of order 12 in the original list.

Since group 1 is non-abelian it must be either 22, 23 or 24. Gather some information to decide which it is.

Aside:
It is interesting to look at the list of permutations in this group and compare each element with its order.

A	()	B	(2 3 4)	C	(2 4 3)
D	(1 2)(3 4)	E	(1 2 3)	F	(1 2 4)
G	(1 3 2)	H	(1 3 4)	I	(1 3)(2 4)
J	(1 4 2)	K	(1 4 3)	L	(1 4)(2 3)

```
Group number 1 of Order 12
    1 elements of order  1:   A
    3 elements of order  2:   D I L
    8 elements of order  3:   B C E F G H J K
    0 elements of order  4:
    0 elements of order  6:
    0 elements of order 12:
```

```
G1>> ORDERS   for Group Number 22

Group number 22 of Order 12
    1 elements of order  1:   A
    7 elements of order  2:   D G H I J K L
    2 elements of order  3:   C E
    0 elements of order  4:
    2 elements of order  6:   B F
    0 elements of order 12:
```

Now compare the orders of elements for groups 22 and 23 with our permutation group.

```
G22>> ORDERS   for Group Number 23

Group number 23 of Order 12
    1 elements of order  1:   A
    3 elements of order  2:   D I K
    8 elements of order  3:   B C E F G H J L
    0 elements of order  4:
    0 elements of order  6:
    0 elements of order 12:

G23>> ORDERS   for Group Number 1

Group number 1 of Order 12
    1 elements of order  1:   A
    3 elements of order  2:   D I L
    8 elements of order  3:   B C E F G H J K
    0 elements of order  4:
    0 elements of order  6:
    0 elements of order 12:
```

We attempt to find an isomorphism between group 1 and group 32. Depending on how your version of Groups32 is set, this either takes place in a separate window or else erases the current worksheet.

If your version uses a separate window, the window will close if you have success. The result can be inserted into the current worksheet using the RESULT command.

```
G1>> ISOMORPHISM   from Group Number 1 to Group Number 23

G1>> RESULT
        Top: Group # 1          BOTTOM: Group # 23

        A B C D E F G H I J K L
        A B C D F E G H I J L K
```

Here are steps in finding the isomorphism:

Top: Group # 1 BOTTOM: Group # 23

A B C D E F G H I J K L

Send:

Top: Group # 1 BOTTOM: Group # 23

A B C D E F G H I J K L

Send: B To: B

Some elements remain to be mapped

```
Top: Group # 1           BOTTOM: Group # 23

A B C D E F G H I J K L
A B C

Redo the last assignment (y/n/q)?

Some elements remain to be mapped
```

Top: Group # 1 BOTTOM: Group # 23

A B C D E F G H I J K L
A B C

Send: D To: D

Some elements remain to be mapped

Top: Group # 1 BOTTOM: Group # 23

A B C D E F G H I J K L
A B C D F E G H I J L K

Send: D To: D

*** SUCCESS ***

```
G1>> QUIT
```

This has been a quick tour of some of the commands in Groups32. Remember that you can always get a list of commands by typing "H" and you can always get a description of a command by typing "X" or "INFO" first. You should now be able to explore on your own.

CENTER	CENTRALIZER	CHART	CONJ-CLS
COSETS	EVALUATE	EXAMPLES	GENERATE
GROUP	HELP	INFO	ISOMORPHISM
LEFT	NORMALIZER	ORDERS	PERMGRPS
POWERS	QUIT	RESULT	RIGHT
SEARCH	STOP	SUBGROUPS	TABLE
X			

CENTER

The center of the group G is the set of all x so that x*g = g*x for every g in G. It is the set of all elements which commute with everything. You supply the group number (which then becomes the current group). G is abelian if G = Center(G).

CENTRALIZER

The centralizer of an element x of G is the set of all g for which x*g = G*x.

CHART

This prints a chart of all groups of a given order. The groups are numbered 1 to 144 and are grouped by order. An asterisk indicates that the group is not abelian. Input of 0 for the order gives all groups.

CONJ-CLS

This prints the conjugacy classes for the given group.

COSETS

If H is a given subgroup of the current group G, this produces a list of all the distinct left and right cosets of H. H is normal if every right coset is also a left coset.

EVALUATE

This is used to evaluate an expression in the current group. An expression is a collection of group elements and inverses which is evaluated left to right. An apostrophe following a letter is used to indicate the inverse of the letter. Thus BC'D will give the product of B followed by the inverse of C followed by D

EXAMPLES

Several multiplication tables for groups are saved in a disk file. They are obtained using some common ways to describe groups. Select an example and it will be installed to replace group number 1. You can then apply all the built-in operations to this group.

GENERATE

Using the current group, the smallest subgroup containing the given elements is returned.

GROUP

This sets the current group number

HELP

This prints a list of all current commands

INFO

This will provide information about the next command you use. INFO and X do the same thing but X is quicker to use.

ISOMORPHISM

This is a tool for finding isomorphisms between groups. (It will be useful for finding automorphisms of a group and, when additional group tables are installed, for finding isomorphisms between new and existing groups.) The numbers of the groups to be compared must be supplied. You will then see a list of elements of the first group and will choose how an element of the first group is to be mapped to the second. The program will determine all consequences of your mapping -- and let you know if there are inconsistencies. When you are given the choice Redo the last assignment (y/n/q)? you can back up to the previous assignment (selecting y) stick with things as they are (selecting n) or quit from doing the isomorphism (selecting q)

LEFT

Given a group G, subgroup H, and element x of G, The left coset xH is the set of all products x*h where h runs through H.

NORMALIZER

The normalizer of a subgroup H in a group G consists of all elements g so that gHg' = H. It is the largest subgroup of G in which H is a normal subgroup.

ORDERS

This prints a chart showing which elements have which orders in the given group. The group is made the current group.

PERMGRPS

This switches to a package of commands for handling permutations and permutation groups. The command menu is replaced by a menu for this package. You return to the main groups commands by typing MAIN"

Click here to go to the PERMGRPS submenu

POWERS

Prints the powers, x^n for the given element x starting with n = 0 and ending with the highest power not the identity.

QUIT

This will end the Groups32 program.

RESULT*

After you have used ISOMORPHIC and the small secondary window is closed, this command will print the result of the isomorphism in the main window. This can be useful for reference purposes.

* IF YOUR GROUPS32 DOES NOT USE A SECONDARY WINDOW, THIS COMMAND WILL DO NOTHING.

RIGHT

Given a group G, subgroup H, and element x of G, The right coset Hx is the set of all products h*x where h runs through H.

SEARCH

This will search all groups to find those with elements satisfying some requirements you specify. You will be asked for letters giving the elements (at most 5). You can require that these generate the group. You can specify orders. You can specify a list of relations. EXAMPLE: The symmetric group S3 is specified by generators xy with x of order 2, y of order 3, and the relation yx = xy' NOTE: a search can be aborted by pressing ESC
++

STOP

This will end the command interface (but not the groups program). You can resume use of the commands interface by typing 'commands'. *** Exit the program by typing `bye` ***

SUBGROUPS

This produces a list of all the distinct subgroups of the given group (which is made the current group). The subgroups are numbered, a set of generators is given, and then the list of all elements in the subgroup. An asterisk indicates that the subgroup is normal.

TABLE

This prints a table for the group requested (and makes that the current group). Elements are represented by letters A to Z and the symbols [\ ] ^ _ and `

X

This will provide information about the next command you use. INFO and X do the same thing but X is quicker to use.

CREATE	ELEMENTS	HELP	INFO
INSTALL	MAIN	MULTIPLY	QUIT
X			

CREATE

This will determine the subgroup of S_n generated by a given set of permutations (given as a product of cycles). You must put in n (for S_n) and then the generators using numbers 1..n for example (1 2)(3 4 5). The program will only compute groups up to order 51. If the resulting group has order 32 or less, you can install the table as one of the groups 1-5.

ELEMENTS

This will print the list of assignments of letters to permutations for the last groups CREATEd.

INSTALL

If the last generated permutation group has order <=32 it can be installed as one of the group tables 1-5. The facilities of the main group theory package can be applied.

MAIN

Exit the PERMGroups submenu and return to the main menu. Be sure to INSTALL any group that you want to investigate.

MULTIPLY

Find product of cycles in Sn. Example (1 2)(1 3) The result will be = (1 2 3) if this copy of Groups32 has been set to multiply from left to right and (1 3 2) if it multiplies right to left. To quit press ENTER on a blank line.

Running Groups32

You can either click HERE or use Telnet to connect to math.ucsd.edu.

In either case, give "groups32" as username (i.e. login)
and "groups32" (all lower case) as password.