

# COMPUTING ELLIPTIC CURVES OVER COMPLEX CUBIC FIELDS

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## ABSTRACT

In [1], Paul Gunnells and Dan Yasaki computed data on automorphic forms and corresponding elliptic curves over the complex cubic field of discriminant  $-23$ . We complement and extend the results of [1] by doing more investigation of the elliptic curves (e.g., torsion subgroups, rank, and isogenies) and extending the computations to consider other complex cubic fields. Via a variety of methods, we constructed the elliptic curves over the complex cubic field of discriminant  $-23$  corresponding to automorphic forms up to a bound on norm conductor. In particular, assuming a certain notion of modularity, we describe a complete enumeration of elliptic curves over this field with norm conductor less than or equal to 911, the level to which we possess knowledge of automorphic forms. Our enumeration includes the (conjecturally) first elliptic curve of rank 1 over this field when ordering elliptic curves by norm conductor. We also expand the scope of study by computing elliptic curves with small norm conductor over other complex cubic fields to make predictions about the automorphic forms we expect to encounter. Finally, we use these examples to prove several propositions regarding the torsion subgroup possibilities of elliptic curves over these fields, complementing work by Filip Najman in [2].

## REFERENCES

- [1] Paul E. Gunnells and Dan Yasaki, *Modular forms and elliptic curves over the cubic field of discriminant  $-23$* , submitted 2012, <http://arxiv.org/abs/1201.4132v1>.
- [2] Filip Najman, *Torsion of elliptic curves over cubic fields*, J. Number Theory **132** (2012), 26-36, <http://arxiv.org/abs/1108.3709>.

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