

April 17, 2007

Math 140B – Sample Midterm (55 minutes)

Caution: This is only a sample midterm. The real midterm may be quite different!

Instructions. Answer all questions. You may use without proof anything which was proved in class or in the text by Rudin. However, you must reprove items which were given as exercises, such as in 1 and 4 below.

1. Let f be defined for all real x , and suppose that

$$|f(x) - f(y)| \leq |x - y|^{3/2}.$$

Show that $f \equiv 0$. (Not that f is not assumed to be differentiable.)

2. Find constants a_0 and a_1 such that

$$1/(1 - x) = a_0 + a_1x + u(x),$$

where $u(x)$ is a function satisfying $\lim_{x \rightarrow 0} u(x)/x = 0$. Explain why your solution is correct.

3. Prove the following theorem: If $f : (a, b) \rightarrow \mathbb{R}$ is monotonic, then the set of discontinuities is either finite or countably infinite. (This is proved in Rudin, but you should give a proof here.)

4. Suppose that $f : [a, b] \rightarrow \mathbb{R}$ has continuous derivative f' on $[a, b]$. Show that for any $\epsilon > 0$ there exists $\delta > 0$ such that for $t \in [a, b]$,

$$|t - x| < \delta \implies \left| \frac{f(t) - f(x)}{t - x} - f'(x) \right| < \epsilon$$

5. Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable and there exists $M > 0$ such that $|f(x)| \leq M$ for all x .

(a) Show that the function $h(x) := x^2 f(1/x)$ is differentiable at all points.

(b) Suppose that f is as above and also satisfies $f(n) = 1$ for all n even and $f(n) = -1$ for all n odd. (Values at other points are not given.) Show that $\lim_{x \rightarrow 0} f'(1/x)$ does not exist.