Math 140, Winter **Practice problems** **February, 2010**
Instructor: Professor Ni

1. Exercise 8 of Ch4.

2. Exercise 14 of Ch4.

3. Exercise 18 of Ch4.

4. Exercise 19 of Ch4.

5. Find the limit

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{(1+x)^{1/3}-(1-x)^{1/3}}.$$

6. Assume that the sequence $\{a_n\}$ is convergence and $a_n > 0$. Prove that

$$\lim_{n\to\infty}(a_1\cdot a_2\cdot\dots\cdot a_n)^{1/n}=\lim_{n\to\infty}a_n.$$

7. Assume that $\{a_n\}$ satisfies that $0\le a_{n+m}\le a_n+a_m$. Prove that $\{\frac{a_n}{n}\}$ converges.

8. For any true statement below, prove it. For any false statement below find an example, or prove it false if you prefer.

   (i) $\liminf_{n\to\infty}x_n+\liminf_{n\to\infty}y_n\le\liminf_{n\to\infty}(x_n+y_n)\le\liminf_{n\to\infty}x_n+\limsup_{n\to\infty}y_n$.

   (ii) $\liminf_{n\to\infty}x_n+\limsup_{n\to\infty}y_n\le\limsup_{n\to\infty}(x_n+y_n)\le\limsup_{n\to\infty}x_n+\limsup_{n\to\infty}y_n$.

   (iii) Assume that $x_n, y_n\ge 0$. Then $\liminf_{n\to\infty}x_n\cdot\liminf_{n\to\infty}y_n\le\liminf_{n\to\infty}(x_n\cdot y_n)\le\liminf_{n\to\infty}x_n\cdot\limsup_{n\to\infty}y_n$.

   (iv) Assume that $x_n, y_n\ge 0$. Then $\liminf_{n\to\infty}x_n\cdot\limsup_{n\to\infty}y_n\le\limsup_{n\to\infty}(x_n\cdot y_n)\le\limsup_{n\to\infty}x_n\cdot\limsup_{n\to\infty}y_n$.

9. Prove that if for a nonnegative sequence $\{a_n\}$ it holds that for any sequence $\{b_n\}$,

$$\limsup_{n\to\infty}(a_n+b_n)=\limsup_{n\to\infty}a_n+\limsup_{n\to\infty}b_n$$

   and

$$\limsup_{n\to\infty}(a_n\cdot b_n)=\limsup_{n\to\infty}a_n\cdot\limsup_{n\to\infty}b_n.$$

   then $\{a_n\}$ must converges.

10. For a sequence $\{a_n\}$ with $a_n>0$, if

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=a$$

   then

$$\lim_{n\to\infty}a_n^{\frac{1}{n}}=a.$$

   You need to show that $\{a_n^{\frac{1}{n}}\}$ converges.

11. Prove that for function $f(x)$ defined on $[a,+\infty)$, satisfying that $f(x)$ is bounded on any finite (a,b). Then

$$\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}f(x+1)-f(x).$$

   Here we assume that both limit exist. You only need to show that they are the same.