Math 140, Winter **Practice problems** **April, 2010**
Instructor: Professor Ni

1. Page 114: Exercise 4, Exercise 6.
2. Page 115: Exercise 11, 15.
3. Page 117: Exercise 22.
4. Page 138: Exercise 5.
5. Let $f(x) = 1 - x^{\frac{2}{3}}$. Check that $f(-1) = f(1) = 0$. Explain why the mean value theorem fails to conclude that there exists $c \in (-1, 1)$ such that $f'(c) = 0$.
6. Assume that f and its derivatives $f^{(j)}$ are continuous on $[x_0, x_n]$ for all $j \leq n-1$ and $f^{(n)}$ exists on (x_0, x_n). Assume that there exist x_j with $x_0 < x_1 < x_2 \cdots < x_{n-1} < x_n$ such that
$$f(x_0) = f(x_1) = \cdots f(x_n).$$
Prove that there exists $c \in (x_0, x_n)$ such that $f^{(n)}(c) = 0$.
7. Let f is a function such that $f^{(j)}$ for $j \leq p+q$ exist on $[a, b]$ and $f^{(p+q+1)}$ exists on (a, b) and
$$f(a) = f'(a) = \cdots f^{(p)}(a) = 0$$
and
$$f(b) = f'(b) = \cdots = f^{(q)}(b) = 0.$$
Prove that there exists $c \in (a, b)$ such that $f^{(p+q+1)}(c) = 0$.
8. Assume that φ is a monotonically increasing function and differentiable. Assume further that for $x \geq x_0$, $|f'(x)| \leq \varphi'(x)$. Prove that for $x \geq x_0$
$$|f(x) - f(x_0)| \leq \varphi(x) - \varphi(x_0).$$
9. Page 138: 4, 5, 6, 7, 8
10. Prove Theorem 6.12 on page 128.
11. Assume that α is monotonically increasing and $\alpha' \in \mathcal{R}$, namely Riemann integrable, on $[a, b]$. Prove that for any bounded function f prove that
$$\overline{\int_a^b} f(x) d\alpha = \overline{\int_a^b} f(x)\alpha'(x)\, dx.$$