Practice Final Examination Math 21C Fall 1999

1. (20 Points)Give parametric equations for the line of intersection of the plane $x+3y+z+1=0$ and the plane containing the points $(1,1,0),(1,1,1),(2,0,1)$.

2. (20 Points)Sketch the surface defined by the equation $x^2+4y^2+z^2+2x+4y+9z+3=0$.

3. (25 Points)Which of the following functions is continuous at $(0,0)$? (You must give reasons to get full credit.)
a) $f(x,y)=\frac{xy}{1+x^2y^2}$.
b) $f(x,y)=\frac{x^2+2y^2}{x^2+y^2}$ if $(x,y)\neq 0$, $f(0,0)=0$.
c) $f(x,y)=\frac{x^4+3y^3}{x^2+y^2}$ if $(x,y)\neq 0$, $f(0,0)=0$.

4. (20 Points)Give an equation for the tangent plane of the surface $z=x^2-y^2+4$ at the point $(2,3,-1)$.

5. (15 Points)A mountain climber is climbing a "mountain" that is given by the equation $z=x^2+y^2$. Using his compass he knows that he is at the point $(1,1,2)$. In which direction should he head to ascend the fastest?

6. (20 Points)Classify the critical points of the function

$$f(x,y)=x^4-2*x^2+y^4-2*y^2.$$

7. (20 Points)Find the maximum and minimum value of the function $f(x,y)$ of the previous problem in the set $x^2+y^2\leq 16$.

8. (25 Points)Calculate the following double integrals.
a) The integral of $f(x,y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ over the rectangle $1\leq x\leq 3$, $0\leq y\leq 1$.
b) The integral of $f(x,y)=xy$ over the triangle with vertices $(0,0),(0,1),(1,1)$.
c) The integral of $f(x,y)=x+y$ over the region between the curves $y=x^2+1, y=2-x^2$.

9. (15 Points)Calculate the integral of $f(x,y)=e^{x^2+y^2}$ over the set $x^2+y^2<16$. (Hint: Try polar coordinates.)

10. (20 Points)Calculate the following triple integrals.

a) The integral of $f(x, y, z) = 2x - 3y + z$ over the box $0 \leq x \leq 1, -1 \leq y \leq 1, 1 \leq z \leq 3$.

b) The integral of $f(x, y, z) = xyz$ over the three dimensional domain given by $x^2 + y^2 < 4$, $x \geq 0, y \geq 0, 0 \leq z \leq 1$.