Zero Biasing and Combinatorial Central Limit Theorems

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Abstract

For a mean zero, variance one random variable W, we say W^* has the W zero biased distribution if

$$EWf(W) = Ef'(W^*) \quad \text{for all smooth } f.$$

It can be shown using Stein's method, that W, with distribution F, is close to normal when it can be coupled closely to its zero biased version W^* with distribution F^*, as quantified by the L^1 norm inequality

$$||F - \Phi||_1 \le 2||F^* - F||_1,$$

where Φ is the cumulative standard normal. The bound provides Berry Esseen type inequalities, with explicit, moderate, constants. By the use of smoothing inequalities, L^∞ bounds can also be derived in terms of distances between W and W^*. Bounds of these types will be illustrated to assess the quality of the normal approximation in combinatorial central limit theorems, that is, to

$$Y = \sum_{i=1}^{n} a_{i\pi(i)}$$

when the random permutation π has distribution uniform over the symmetric group $\mathcal{S}_n$, and also for certain distributions on $\mathcal{S}_n$ which are constant on conjugacy classes.