ABOUT THE DISTRIBUTION OF PRIMES IN POLYNOMIAL FORM

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ABSTRACT

Define a Bouniakowsky polynomial as an irreducible polynomial $f(x)$ with integer coefficients, degree >1 , and $GCD(f(1), f(2)...) = 1$. The Bouniakowsky conjecture states that $f(x)$ is prime for an infinite number of integers (Bouniakowsky 1857).This paper is a brief study of the distribution of primes in polynomial form. We begin with a brief study of the Sieve of Eratosthenes and recall some basic notions of number theory. We define a generalization of Euler's toitient function, which will give some properties about prime numbers that divide a given polynomial. We will describe in detail an algorithm that generates all the primes of polynomial form and this will make some estimates of the number of primes of a polynomial form using ideas similar to those created by Legendre to estimate the size of $\pi(x)$ from the sieve of Eratosthenes. We will give some estimates of the number of primes of a polynomial form, emphasizing the conjecture $x^2 + 1$, and give necessary and sufficient conditions for a polynomial to generate infinte primes. These conditions make use of the notion of ε-uniformity which will be defined in the text. Finally, since we could not show that any polynomial comply with such conditions, we will show some numerical evidence indicating the feasibility of such compliance.

References

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