Beyond universality in random matrix theory

Abstract:

In random matrix theory, we study the histogram of eigenvalues of matrices sampled from given distributions on their entries, asymptotically as the dimension grows. The (computationally) nicest case – independent Gaussian entries – was initiated by Wigner in the 1950s, and by the 1980s this case was pretty completely understood.

In the last decade, the subject has exploded. Most attention has thus far been focused on two natural broad generalizations of the Gaussian case: either to matrices with independent entries sampled from some other distribution, or matrices whose joint density of entries has a log-concave potential. In both scenarios, we now know the asymptotic behavior is ”universal”: the limit law of eigenvalues, their fluctuations around this limit, and finer statistics like asymptotic spacing and edge behavior, are all the same essentially independent of the specific model used.

In this talk I will discuss the main story of random matrices as we know it today, and also discuss a very new direction: a third broad generalization of ”Gaussian” matrices. Thinking of a Gaussian as the solution to the heat equation on a linear space of matrices, I have been studying the eigenvalues of heat kernel distributed random matrices on Lie groups. As we will discuss, some of the ”universal” behavior from the studied models persists (like the size and form of the fluctuations), but in most cases, these models yield completely new asymptotics that we are only beginning to understand.