Instructions

1. No calculators or other electronic devices are allowed during this exam.
2. You may use one page of notes, but no books or other assistance during this exam.
3. Write your Name, PID, and Section on the front of your Blue Book.
4. Write the Version of your exam on the front of your Blue Book.
5. Write your solutions clearly in your Blue Book
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start each question on a new side of a page.
6. Read each question carefully, and answer each question completely.
7. Show all of your work; no credit will be given for unsupported answers.

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions given during the exam.

1. (6 points) Given the $4 \times 5$ matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 3 \\ 2 & 4 & 1 & -3 & 1 \\ 1 & 2 & 4 & 2 & 1 \\ 3 & 6 & 5 & -1 & 2 \end{bmatrix}$.

   $A$ is row equivalent to the matrix $B = \begin{bmatrix} 1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   Find a basis for each of the following subspaces associated with the matrix $A$:

   (a) (1 point) $\text{Col}(A)$, the column space of $A$.
   (b) (1 point) $\text{Row}(A)$, the row space of $A$.
   (c) (3 points) $\text{Nul}(A)$, the null space of $A$.
   (d) (1 point) $\text{Col}(A^T)$, the column space of $A^T$.

Exam continues with Problems 2 – 4 on the other side of this sheet.
2. (6 points) Given the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$. $A$ can be row-reduced to the identity matrix with a sequence of three elementary row operations as follows:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) Write down the elementary matrices $E_1$, $E_2$, $E_3$ corresponding to the elementary row operations 1, 2, 3 indicated above.

(b) Express $A^{-1}$ as a product of elementary matrices.

(c) Compute $A^{-1}$.

3. (6 points) Let $A$ be a $m \times n$ matrix and $B$ a $n \times p$ matrix. Suppose that the $m \times p$ matrix $C = AB$ has the property that $Cx = 0$ has only the trivial solution. Find $\dim \text{Col}(B)$, the dimension of the column space of $B$. Be sure to justify your answer.

4. (6 points) Given $U$ and $W$ subspaces of a vector space $V$. Is the union $U \cup W$ a subspace of $V$? Why or why not?