

A Riemann-sum program

TI-83 and TI-83 plus calculators

The following program for Texas Instruments TI-83 and TI-83 plus calculators draws the rectangles associated with Riemann sums for $\int_a^b f(x) dx$ with $f(x)$ entered as $Y_1(X)$ in the $Y=$ menu and gives the values of the Riemann sums. The user enters $a = A$, $b = B$, the number N of subintervals, and a number T between 0 and 1. The rectangles are of equal widths, and the program gives the left Riemann sum if $T = 0$, the right Riemann sum if $T = 1$, and the midpoint Riemann sum if $T = 0.5$.

1.	PROGRAM: RIEMANN	13.	: For($J, 1, N, 1$)
2.	: ClrDraw	14.	: $A + (J - 1)D \rightarrow U$
3.	: FnOff	15.	: $U + TD \rightarrow X$
4.	: FnOn 1	16.	: $Y_1 \rightarrow W$
5.	: Disp "USE $Y_1(X)$ "	17.	: Line(U, \emptyset, U, W)
6.	: Prompt A, B, N	18.	: Line($U, W, U + D, W$)
7.	: Disp " $T = \emptyset$ FOR LEFT"	19.	: Line($U + D, W, U + D, \emptyset$)
8.	: Disp " $T = 1$ FOR RIGHT"	20.	: $S + DW \rightarrow S$
9.	: Disp " $T = 0.5$ FOR MID"	21.	: End
10.	: Input T	22.	: Pause
11.	: $(B - A)/N \rightarrow D$	23.	: Disp "SUM ="
12.	: $\emptyset \rightarrow S$	24.	: Disp S

The symbol \emptyset in lines 7, 9, 12, 17, and 19 of the program denotes zero. Be careful to use $-$ and not $(-)$ for the subtraction in lines 11 and 14.

Transferring the program

To transfer the program electronically from one TI-83 calculator to another, turn both calculators on and connect them with the wire that is provided, pressing the connectors in firmly. On the calculator that is to receive the program, press **2nd** **LINK**, select \langle RECEIVE \rangle , and press **ENTER**. The screen should display "Waiting. . ." On the transmitting calculator press **2nd** **LINK**, scroll down to \langle Prgm \rangle , press **ENTER**, and scroll down to the name of the program. Press **ENTER** to select it. The triangle beside its name should change to a square. Press **▶** for \langle Transmit \rangle and **ENTER** to transmit the program. Select \langle Overwrite \rangle on the receiving calculator if necessary to replace any old program that has the same name.

Running the program

Before this program is run, the function to be studied should be entered as Y_1 in the $Y=$ menu and a suitable window for the graph of the function should be chosen. To run the program, press **PRGM**, scroll down to the program's name, and press **ENTER**. Give the limits of integration, A and B , the number of subintervals, N , and T ($= 0, 1$ or 0.5) when requested, followed each time by the **ENTER** key. The curve and the rectangles will be drawn. Press **ENTER** to display the value of the Riemann sum. You can then press **ENTER** to rerun the program if you do not perform any other operations first.

Example 1 (a) Use the Riemann-sum program to calculate the mid-point Riemann sums for $\int_0^1 x^2 dx$ corresponding to the partition of $[0, 1]$ into 5, 10, 20, 50, and 100 equal subintervals. (b) Use the results to predict the exact value of the integral.

SOLUTION

(a) Enter $Y_1 = x^2$ in the $Y=$ menu and set the window to be $-0.25 \leq x \leq 1.25, -0.25 \leq y \leq 1.25$. Press $\boxed{\text{PRGM}}$, the program's number, and $\boxed{\text{ENTER}}$. Enter $A = 0, B = 1, N = 5$, and $T = 0.5$ when requested. The rectangles for the Riemann sum with five subintervals will be shown with the curve, as in Figure 1. Press $\boxed{\text{ENTER}}$ for the value 0.33 of the Riemann sum. Press $\boxed{\text{ENTER}}$ and rerun the program with the other values of N to obtain Figures 2 and 3 for $N = 10$ and 20, and the values in the table for $N = 10, 20, 50$ and 100.

(b) The Riemann sums appear to be approaching $0.333\cdots = \frac{1}{3}$, which would then be the value of the integral. \square

N	5	10	20	50	100
Riemann sum	0.33	0.3325	0.333125	0.3333	0.333325

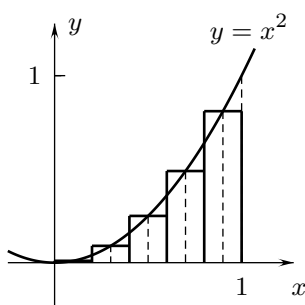


FIGURE 1

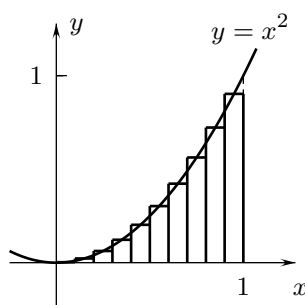


FIGURE 2

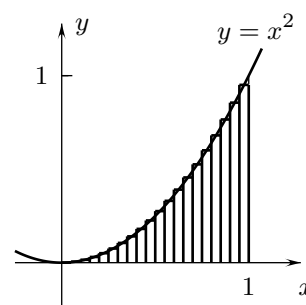


FIGURE 3

Entering the program

Press $\boxed{\text{MODE}}$ and put the calculator in function mode (select $\langle \text{Func} \rangle$ on line 4) if necessary. Press $\boxed{2\text{nd}} \boxed{\text{QUIT}}$ to return to the home screen, press $\boxed{\text{PRGM}} \boxed{\blacktriangleright} \boxed{\blacktriangleright}$ for $\langle \text{NEW} \rangle$, press $\boxed{\text{ENTER}}$, and execute the following line-by-line instructions to enter the program.

To make a correction, move the cursor to the error and use $\boxed{\text{INS}}$ and $\boxed{\text{DEL}}$ as needed. To return to the program from other screens, press $\boxed{\text{CLEAR}}$ or follow the steps in the previous paragraph. To interrupt the running of the program, press $\boxed{\text{ON}}$ and then $\boxed{2}$ for QUIT.

To delete a program, press $\boxed{2\text{nd}} \boxed{\text{MEM}} \boxed{2}$ for Delete, $\boxed{6}$ for Prgm, put the cursor on the name of the program, and press $\boxed{\text{ENTER}} \boxed{2\text{nd}} \boxed{\text{QUIT}}$.

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1. Press $\boxed{\text{R}} \boxed{\text{I}} \boxed{\text{E}} \boxed{\text{M}} \boxed{\text{A}} \boxed{\text{N}} \boxed{\text{N}} \boxed{\text{ENTER}}$. The calculator is automatically in alpha mode here. This title identifies the program.
 2. Press $\boxed{2\text{nd}} \boxed{\text{DRAW}} \boxed{1} \boxed{\text{ENTER}}$. ClearDraw clears the graphics screen.
 3. Press $\boxed{\text{VARS}} \boxed{\blacktriangleright} \boxed{4} \boxed{2} \boxed{\text{ENTER}}$ to unselect all the functions in the $Y=$ menu.
 4. Press $\boxed{\text{VARS}} \boxed{\blacktriangleright} \boxed{4} \boxed{1} \boxed{1} \boxed{\text{ENTER}}$ to select Y_1 .
 5. Press $\boxed{\text{PRGM}} \boxed{\blacktriangleright} \boxed{3}$ for the word Disp and then $\boxed{2\text{nd}} \boxed{\text{A-LOCK}} \boxed{\text{“}} \boxed{\text{U}} \boxed{\text{S}} \boxed{\text{E}} \boxed{-} \boxed{\text{VARS}} \boxed{\blacktriangleright} \boxed{1} \boxed{1} \boxed{(} \boxed{X, T, \theta, n} \boxed{)} \boxed{\text{ALPHA}} \boxed{\text{“}} \boxed{\text{ENTER}}$. The space $\boxed{-}$ is on the key with 0 and $\boxed{\text{VARS}}$ is the $\langle \text{VARS} \rangle$ key.
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6. Press $\boxed{\text{PRGM}} \boxed{\blacktriangleright} \boxed{2} \boxed{\text{ALPHA}} \boxed{A} \boxed{,} \boxed{\text{ALPHA}} \boxed{B} \boxed{,} \boxed{\text{ALPHA}} \boxed{N} \boxed{\text{ENTER}}$ to cause the prompts $A = ?$, $B = ?$, and $N = ?$ to appear when the program is run so the user can supply values of A , B , and N .
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7. Press $\boxed{\text{PRGM}} \boxed{\blacktriangleright} \boxed{3} \boxed{\text{ALPHA}} \boxed{\text{“}} \boxed{\text{ALPHA}} \boxed{T} \boxed{2\text{nd}} \boxed{\text{TEST}} \boxed{1} \boxed{\emptyset} \boxed{2\text{nd}} \boxed{\text{A-LOCK}} \boxed{-} \boxed{\text{F}} \boxed{\text{O}} \boxed{\text{R}} \boxed{-} \boxed{\text{L}} \boxed{\text{E}} \boxed{\text{F}} \boxed{\text{T}} \boxed{\text{“}} \boxed{\text{ENTER}}$. This and the next two lines explain the role of T .
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8. Press $\boxed{\text{PRGM}} \boxed{\blacktriangleright} \boxed{3} \boxed{\text{ALPHA}} \boxed{\text{“}} \boxed{\text{ALPHA}} \boxed{T} \boxed{2\text{nd}} \boxed{\text{TEST}} \boxed{1} \boxed{1} \boxed{2\text{nd}} \boxed{\text{A-LOCK}} \boxed{-} \boxed{\text{F}} \boxed{\text{O}} \boxed{\text{R}} \boxed{-} \boxed{\text{R}} \boxed{\text{I}} \boxed{\text{G}} \boxed{\text{H}} \boxed{\text{T}} \boxed{\text{“}} \boxed{\text{ENTER}}$.
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9. Press $\boxed{\text{PRGM}} \boxed{\blacktriangleright} \boxed{3} \boxed{\text{ALPHA}} \boxed{\text{“}} \boxed{\text{ALPHA}} \boxed{T} \boxed{2\text{nd}} \boxed{\text{TEST}} \boxed{1} \boxed{\emptyset} \boxed{.} \boxed{5} \boxed{2\text{nd}} \boxed{\text{A-LOCK}} \boxed{-} \boxed{\text{F}} \boxed{\text{O}} \boxed{\text{R}} \boxed{-} \boxed{\text{M}} \boxed{\text{I}} \boxed{\text{D}} \boxed{\text{“}} \boxed{\text{ENTER}}$.
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10. Press $\boxed{\text{PRGM}} \boxed{\blacktriangleright} \boxed{1} \boxed{\text{ALPHA}} \boxed{T} \boxed{\text{ENTER}}$. The user picks T .
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11. Press $\boxed{(} \boxed{\text{ALPHA}} \boxed{B} \boxed{-} \boxed{\text{ALPHA}} \boxed{A} \boxed{)} \boxed{\div} \boxed{\text{ALPHA}} \boxed{N} \boxed{\text{STO}} \boxed{\blacktriangleright} \boxed{\text{ALPHA}} \boxed{D} \boxed{\text{ENTER}}$. The width D of the rectangles is calculated and stored.
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12. Press $\boxed{\emptyset} \boxed{\text{STO}} \boxed{\blacktriangleright} \boxed{\text{ALPHA}} \boxed{S} \boxed{\text{ENTER}}$. The Riemann sum is stored as S during the calculations. Here S is set equal to 0 to erase its previous value.
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13. Press $\boxed{\text{PRGM}} \boxed{4} \boxed{\text{ALPHA}} \boxed{J} \boxed{,} \boxed{1} \boxed{,} \boxed{\text{ALPHA}} \boxed{N} \boxed{,} \boxed{1} \boxed{)} \boxed{\text{ENTER}}$. The commands between here and the End on line 21 will be executed for each integer J from 1 to N .
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14. Press $\boxed{\text{ALPHA}} \boxed{A} \boxed{+} \boxed{(} \boxed{\text{ALPHA}} \boxed{J} \boxed{-} \boxed{1} \boxed{)} \boxed{\text{ALPHA}} \boxed{D} \boxed{\text{STO}} \boxed{\blacktriangleright} \boxed{\text{ALPHA}} \boxed{U} \boxed{\text{ENTER}}$. U is the x -coordinate of the left side of the rectangle.
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15. Press $\boxed{\text{ALPHA}} \boxed{U} \boxed{+} \boxed{\text{ALPHA}} \boxed{T} \boxed{\text{ALPHA}} \boxed{D} \boxed{\text{STO}} \boxed{\blacktriangleright} \boxed{X,T,\theta,n} \boxed{\text{ENTER}}$. Here X is the x -coordinate of the left edge of the rectangle if $T = 0$, the right edge if $T = 1$, and the midpoint if $T = 0.5$.
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16. Press $\boxed{\text{VARS}} \boxed{\blacktriangleright} \boxed{1} \boxed{1} \boxed{\text{STO}} \boxed{\blacktriangleright} \boxed{\text{ALPHA}} \boxed{W} \boxed{\text{ENTER}}$. The value at X of the function Y_1 in the $Y=$ menu is calculated and stored as W .
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17. Press $\boxed{2\text{nd}} \boxed{\text{DRAW}} \boxed{2} \boxed{\text{ALPHA}} \boxed{U} \boxed{,} \boxed{\emptyset} \boxed{,} \boxed{\text{ALPHA}} \boxed{U} \boxed{,} \boxed{\text{ALPHA}} \boxed{W} \boxed{)} \boxed{\text{ENTER}}$. The left side of the rectangle is drawn.
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18. Press $\boxed{2\text{nd}} \boxed{\text{DRAW}} \boxed{2} \boxed{\text{ALPHA}} \boxed{U} \boxed{,} \boxed{\text{ALPHA}} \boxed{W} \boxed{,} \boxed{\text{ALPHA}} \boxed{U} \boxed{+} \boxed{\text{ALPHA}} \boxed{D} \boxed{,} \boxed{\text{ALPHA}} \boxed{W} \boxed{)} \boxed{\text{ENTER}}$. The top of the rectangle is drawn.
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19. Press $\boxed{2\text{nd}} \boxed{\text{DRAW}} \boxed{2} \boxed{\text{ALPHA}} \boxed{U} \boxed{+} \boxed{\text{ALPHA}} \boxed{D} \boxed{,} \boxed{\text{ALPHA}} \boxed{W} \boxed{,} \boxed{\text{ALPHA}} \boxed{U} \boxed{+} \boxed{\text{ALPHA}} \boxed{D} \boxed{,} \boxed{\emptyset} \boxed{)} \boxed{\text{ENTER}}$. The right side of the rectangle is drawn.
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20. Press $\boxed{\text{ALPHA}} \boxed{S} \boxed{+} \boxed{\text{ALPHA}} \boxed{D} \boxed{\text{ALPHA}} \boxed{W} \boxed{\text{STO}} \boxed{\blacktriangleright} \boxed{\text{ALPHA}} \boxed{S} \boxed{\text{ENTER}}$. The area of the rectangle is added to S if W is positive and is subtracted from S if W is negative.
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21. Press `PRGM` `7` `ENTER`. The loop starting on line 13 ends here.

22. Press `PRGM` `8` `ENTER`. The program pauses to show the graph and rectangles.

23. Press `PRGM` `▶` `3` `2nd` `A-LOCK` `"` `S` `U` `M` `2nd` `TEST` `1` `ALPHA` `"` `ENTER`. The symbols SUM = are displayed.

24. Press `PRGM` `▶` `3` `ALPHA` `S` `ENTER`. The value of the Riemann sum is displayed.

Press `2nd` `QUIT` to return to the home screen.