(9/7/08)

Math 20C. Lecture Examples.

Sections 15.1 and 15.2. Double integrals†

Example 1 What is the value of $\iint_R \sqrt{9-x^2-y^2}\,dx\,dy$ where R is the disk $x^2+y^2 \le 9$?

Answer: $\iint_R \sqrt{9-x^2-y^2}\,dx\,dy =$ [Volume of the hemisphere in Figure A1] $= 18\pi$

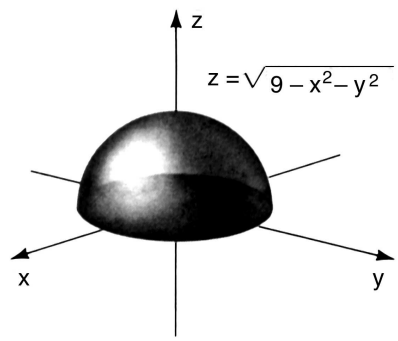


Figure A1

Example 2 Evaluate $\iint_R 2xy\,dx\,dy$, where R is the region bounded by the curve $y = x^2$ and the lines $y = 0$ and $x = 2$ by performing a y-integration first.

Answer: Figure A2 • $\iint_R 2xy\,dx\,dy = \frac{32}{3}$

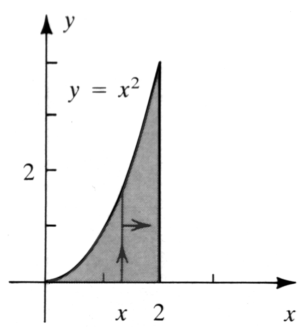


Figure A2

Example 3 Evaluate the integral of Example 2 by performing an x-integration first.

Answer: Figure A3 • $\iint_R 2xy\,dx\,dy = \frac{32}{3}$

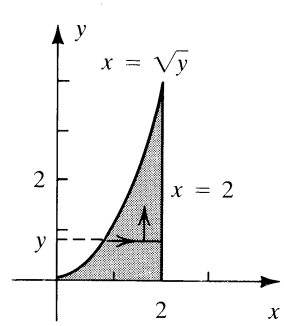


Figure A3

†Lecture notes to accompany Sections 15.1 and 15.2 of *Calculus, Early Transcendentals* by Rogawski.

Example 4 Evaluate $\displaystyle\int_0^8 \int_{x^{1/3}}^2 \sin(y^4)\, dy\, dx$ by reversing the order of integration.

Answer: Figures A4a and A4b • $\displaystyle\int_0^8 \int_{x^{1/3}}^2 \sin(y^4)\, dy\, dx = \int_0^2 \int_0^{y^3} \sin(y^4)\, dx\, dy = \tfrac{1}{4} - \tfrac{1}{4}\cos(16)$

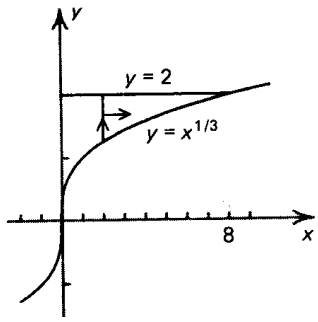


Figure A4a

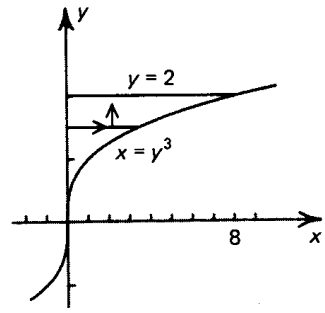


Figure A4b

Example 5 What is the volume of the solid V bounded by the surface $z = \sin x \sin y$ and the plane $z = -2$ for $0 \le x \le \pi, 0 \le y \le \pi$?

Answer: The projection of the solid on the xy-plane is the square in Figure A5. • [Volume] $= 4 + 2\pi^2$

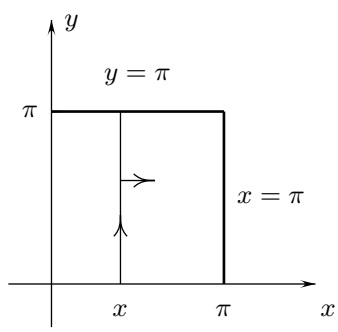


Figure A5

Example 6 A plate that occupies the region $R : 0 \le x \le 2, 0 \le y \le x^2$ with distances measured in feet has density $\rho(x, y) = 3xy^2$ pounds per square foot at (x, y). How much does the plate weigh?

Answer: Figure A6 • [Weight] $= \displaystyle\iint_R 3xy^2\, dx\, dy = 32$ pounds

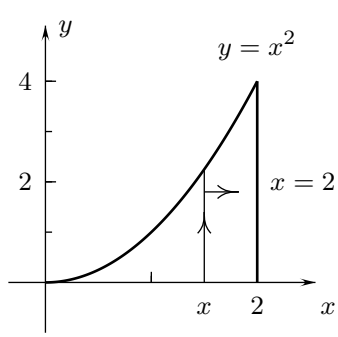
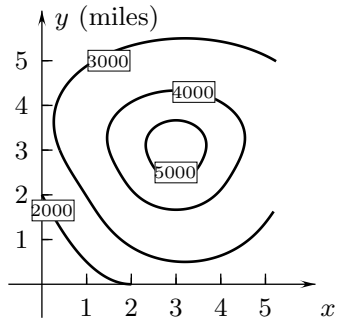


Figure A6

Example 7 Figure 1 shows level curves of the population density $z = p(x, y)$ (people per square mile) in a city. Find the approximate population in the region R: $0 \le x \le 4$, $0 \le y \le 4$. Use a Riemann sum corresponding to a partition into four squares.



Level curves of $z = p(x, y)$
FIGURE 1

Answer: [Population] $= \iint_R p(x, y)\, dy\, dx$ • One approach: Approximate the integral with the Riemann sum corresponding to the partition of R into four equal square subregions of Figure A7 with the population density evaluated at their midpoints. • The population is approximately 58,000.

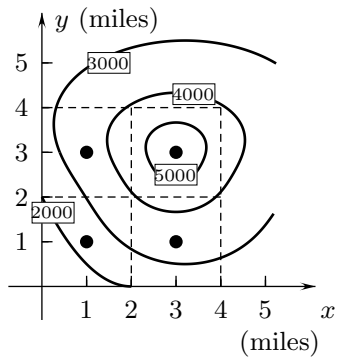


Figure A7

Example 8 What is the average value of $f(x, y) = ye^{y^2}$ on the region $R : 0 \le y \le \sqrt{x}, 0 \le x \le 1$?

Answer: Figure A8 • [Area of R] $= \frac{2}{3}$ • $\iint_R ye^{y^2}\, dx\, dy = \frac{1}{2}(e - 2)$ • [Average value] $= \frac{3}{4}(e - 2)$

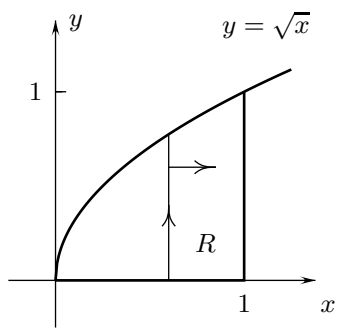


Figure A8

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 16.1: Examples 1 through 5

Section 16.2: Examples 1, 2a, and 3

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.