Problem 6.6. We are looking for integral solution for \(x + y + z = 12\) such that \(0 \leq x \leq 6, 0 \leq y \leq 6, 0 \leq z \leq 3\). Let \(S\) denote all solution with \(x \geq 0, y \geq 0, z \geq 0\). Number of such solution is \(\binom{14}{2}\). Let \(A_1\) be the set of these solutions with \(x \geq 7\). Then \(|A_1| = \binom{7}{2}\). Let \(A_2\) be the set of solution with \(y \geq 7\), then \(|A_2| = \binom{7}{2}\). And let \(A_3\) be the set of solutions with \(z \geq 4\), which implies \(|A_3| = \binom{10}{2}\). Hence \(|A_1 \cap A_2| = 0\), \(|A_2 \cap A_3| = \binom{3}{2}\) and \(|A_1 \cap A_3| = \binom{3}{2}\). Finally \(|A_1 \cap A_2 \cap A_3| = 0\). By the inclusion exclusion principle, our answer is

\[
\left(\binom{14}{2} - \binom{7}{2} - \binom{7}{2} + \binom{3}{2} + \binom{3}{2}\right) = 10.
\]

Problem 6.12. \(\binom{8}{4}D_4\).

Problem 6.15. (a) \(D_7\). (b) \(7! - D_7\). (c) \(7! - 7D_6 - D_7\).

Problem 6.17. Let \(S\) be the set of all permutations. Then we have \(|S| = \frac{9!}{3!2!2!}\). Let \(A_1\) be the number of these permutations with \(aaa\). Then \(|A_1| = \frac{7!}{3!2!2!}\). Let \(A_2\) be the permutation with \(bbbb\). Then we have \(|A_2| = \frac{6!}{3!2!}\). Finally let \(A_3\) be the permutations with \(cc\). Then \(|A_3| = \frac{5!}{3!}\). Hence \(|A_1 \cap A_2| = \frac{4!}{2!}\), \(|A_2 \cap A_3| = \frac{4!}{2!}\) and \(|A_1 \cap A_3| = \frac{5!}{3!}\). Finally we have \(|A_1 \cap A_2 \cap A_3| = 3\). By the inclusion exclusion principle, our answer is

\[
\frac{9!}{3!4!2!} - \frac{7!}{4!2!} - \frac{6!}{3!2!} - \frac{8!}{3!4!} + \frac{4!}{2!} + \frac{6!}{4!} + \frac{5!}{3!} - 3.
\]

Problem 6.18. \((n - 1)! = (n - 1) \cdot (n - 2)!\).

Problem 6.19.

\[
D_n - (n - 1)(D_{n-1} + D_{n-2}) = n! \sum_{i=0}^{n} \frac{(-1)^i}{i!} - (n - 1)(n - 2)! \sum_{i=0}^{n-2} \frac{(-1)^i}{i!} - (n - 1)(n - 1)! \sum_{i=0}^{n-1} \frac{(-1)^i}{i!}.
\]

combining like terms gives us 0.