Problem 1. How many sets of three integers between 1 and 20 are possible if no two consecutive integers are to be in a set?

Problem 2. A ferris wheel has five cars, each containing four seats in a row. There are 20 people ready for a ride. In how many ways can the ride begin? What if a certain two people want to sit in different cars?

Problem 3. A collection of subsets of \( \{1, 2, \ldots, n\} \) has the property that each pair of subsets has at least one element in common. Prove that there are at most \( 2^{n-1} \) subsets in the collection.

Problem 4. Prove that the Ramsey number \( r(3, 3, 3, 3) = r(K_3, K_3, K_3, K_3) \leq 66 \).

Problem 5. Consider a 3-dimensional grid whose dimensions are 10 by 15 by 20. You are at the front lower left corner of the grid and wish to get to the back upper right corner 45 “blocks” away. How many different routes are there in which you walk exactly 45 blocks.

Problem 6. Evaluate \( \sum_{k=0}^{n} 2^k 3^{n-k} \binom{n}{k} \).

Problem 7. A footrace takes place among four runners. If ties are allowed (even all four runners finishing at the same time), how many ways are there for the race to finish?

Problem 8. Prove the Erdős-Szekeres monotone subsequence theorem.