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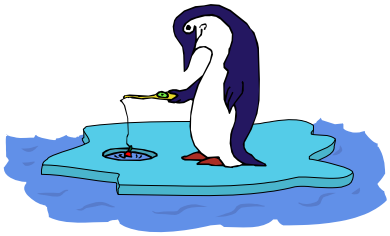
Section Time:_____



Math 20D- Practice Midterm Exam 2 – May 16, 2008

The exam is closed book, no calculators, no computers, no notes, no headphones

Each problem is worth the same number of points. Write the answers on this exam.

1) Use the method of reduction of order to find a second solution of the given differential equation:

$t^2y''+2ty'-2y=0,\ t>0,\ y_1(t)=t$

2) Use the method of undetermined coefficients to solve $y''+y'-2y=2t;\ \ y(0)=0, y'(0)=1.$

3) True - False. Tell whether the following statements are true or false. Give a brief reason for your answer.

a) The ODE $y''+py'+qy=0$ can have 3 linearly independent solutions.

b) The Wronskian determinant of 2 solutions of the ODE in part a) satisfies a first order separable ODE.

c) Every 2x2 matrix has 2 eigenvalues and 2 linearly independent eigenvectors

4) Verify that the given functions y1 and y2 satisfy the corresponding homogeneous equation. Then find a particular solution of the given nonhomogeneous equation.

$t^2y''-2y=3t^2-1,\ t>0,\ y_1(t)=t^2,\ y_2(t)=t^{-1}.$

5) Transform the given initial value problem into an initial value problem for a system of 2 1st order equations.

$u''+u'+4u=\cos(3t),\ \ u(0)=1,\ \ u'(0)=-1.$

6) Check that the given vector satisfies the given differential equation:

$$\vec{x}'=\begin{pmatrix}1 & 1 & 1\\ 2 & 1 & -1\\ 0 & -1 & 1\end{pmatrix}\vec{x}, \qquad \vec{x}=\begin{pmatrix}6\\ -8\\ -4\end{pmatrix}e^{-t}+2\begin{pmatrix}0\\ 1\\ -1\end{pmatrix}e^{2t}.$$

7) Find all the eigenvalues and eigenvectors of the matrix $\begin{pmatrix}-2 & 1\\ 1 & -2\end{pmatrix}$.

8) Solve $\vec{x}'=\begin{pmatrix}-2 & 1\\ 1 & -2\end{pmatrix}\vec{x}$

9) A mass on a spring with spring constant k satisfies the differential equation $mu''+ku=0$, where u(t) is the displacement of the mass from equilibrium at time t.

a) Show that the corresponding system of differential equations is $\vec{x}'=\begin{pmatrix}0 & 1\\ -\frac{k}{m} & 0\end{pmatrix}\vec{x}$

b) Find the eigenvalues of the matrix in this system.

c) Find a fundamental set of solutions of the system.