1) Assume that F is a field. Prove that every polynomial in F[x] factors uniquely into irreducibles up to order and units.

2) a) Find g.c.d.(x^2+1, x^4+x^3+x^2+1) in \( \mathbb{Z}_2[x] \).

   b) Factor x^3+1 as a product of monic irreducible polynomials in \( \mathbb{Z}_7[x] \).

   c) Write x^4+x^3+x^2+1 as a product of monic irreducible polynomials in \( \mathbb{Z}_2[x] \).

3) a) Assume p=prime. How many irreducible polynomials of the form f(x)=x^2+ax+b are there in \( \mathbb{Z}_p[x] \) ?

   b) Show that for every prime p, there exists a field with p^2 elements.

4) a) Find all zeros of \( f(x)=3x^2+x+4 \) in \( \mathbb{Z}_7 \) by the process of substituting all elements of \( \mathbb{Z}_7 \).

   b) Find all zeros of the polynomial f(x) in part a) using the quadratic formula for \( \mathbb{Z}_7 \). Do your answers agree? Should they?

   c) Same as a) for \( g(x)=2x^2+x+3 \) over \( \mathbb{Z}_5 \).

   d) Same as b) for \( g(x) \) in part c).

   e) State necessary and sufficient conditions for the quadratic formula to work for \( \mathbb{Z}_p \), where p=prime. How is this result analogous to what happens with the quadratic formula over the field of real numbers?