∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε

Math 140c – Homework #4 - due Thurs. May 7
(or in drop box by noon of the next day)

∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε

1) Define f(x,y)=1 if $(x,y)=\left(\frac{p}{2^n},\frac{q}{2^n}\right),\ 0<p,q<2^n$, where p,q and n are positive integers. Otherwise define f(x,y)=0.

a) Show that if we hold y fixed, then $f_y(x)=f(x,y)$ is Riemann integrable on [0,1].

b) Show that f is not Riemann integrable on the unit square $[0,1]^2$.

2) Find the integral of f(x,y)=x+y over the domain D consisting of the triangle with vertices (0,0), (0,1) and (2,0).

3) Suppose f(x,y)=1 if y is rational and f(x,y)=x, if y is irrational. Show that f is not Riemann integrable on $[0,1]^2$.

4) Explain why Fubini's theorem fails for the following example:

$$\int_0^1\int_0^1 \frac{x^2-y^2}{\left(x^2+y^2\right)^2}\,dy\,dx=\frac{\pi}{4}$$

$$\int_0^1\int_0^1 \frac{x^2-y^2}{\left(x^2+y^2\right)^2}\,dx\,dy=\frac{-\pi}{4}.$$

5) A **negligible set** S in the plane means that $\forall\ \varepsilon>0$, S has a covering by a finite number of rectangles $S\subset R_1\cup\cdots\cup R_m$ such that the sum of the areas $\text{area}(R_1)+\cdots+\text{area}(R_m)<\varepsilon$.

A set S of **Lebesgue measure 0** in the plane means that $\forall\ \varepsilon>0$, S has a covering by a sequence of rectangles

$$S\subset\bigcup_{i=1}^{\infty}R_i\ \ s.t.\ \ \sum_{i=1}^{\infty}area(R_i)<\varepsilon.$$

Show that the rational points in the unit square $\mathbb{Q}^2\cap[0,1]^2$ is measure 0 but not negligible.