∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε

Math 140c – Homework #5 - due Tues. May 19
(in drop box by noon)

∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε

1) Suppose that f is admissible and equal to 0 outside the rectangle S. Prove that $\iint_S f = \iint_R f$ for every rectangle $R \supset S$.

2) Let f be integrable in $a \le x \le b$ and g integrable in $c \le y \le d$. Show that $f(x)g(y)$ is integrable in $[a,b] \times [c,d]$.

3) Use 2) to see that if f and g are integrable on [a,b], then

$$\frac{1}{2}\iint_{[a,b]^2} \left(f(x)g(y) - f(y)g(x)\right)^2 dxdy = \left(\int_a^b f(x)^2 dx\right)\left(\int_a^b g(x)^2 dx\right) - \left(\int_a^b f(x)g(x)dx\right)^2 .$$

This implies the Cauchy Schwarz inequality for the inner product $\int_a^b fg$ on the space of integrable functions on [a,b].

4) (A Mean-Value Theorem for Multiple Integrals) Suppose that $f,g: D \to \mathbb{R}$ are bounded and continuous on the admissible set $D \subset \mathbb{R}^2$. If D is connected and $g(x) \ge 0$, for all $x \in D$, then there is a point $c \in D$ such that

$$\iint_D fg = f(c) \iint_D g .$$

5) Find the volume of the tetrahedron bounded by the coordinate planes and the plane x+y+z=1,

6) Suppose that $g:[a,b] \to \mathbb{R}$ is continuous. Show that the graph of g (i.e., the set $\{(x,g(x)) \mid x \in [a,b]\}$) is a negligible subset of $\mathbb{R}^2$.

7) Evaluate $\int_0^\infty e^{-x^2} dx$ by squaring it and then writing the result as a double integral. Then change to polar coordinates.

∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε