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Math 140c – Homework #6 - due Thurs. June 4 (in drop box by 2 P.M. next day)

∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε∃δ∃∀∂∞∫∀ε

1) Suppose that D is a region in the plane such that Green's theorem holds. Assume that f is harmonic on D; i.e., $\Delta f = f_{xx}+f_{yy}=0$ on D. Show that $\int_{\partial D} f_y dx - f_x dy = 0$.

2) Let $P(x,y) = \frac{-y}{x^2+y^2}$, $Q(x,y) = \frac{x}{x^2+y^2}$. Let $D = \{(x,y) \mid x^2+y^2 \le 1\}$. Show that Green's theorem fails in this case ; i.e., $\int_{\partial D} Pdx + Qdy \neq \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy$? Why does this not contradict what we proved?

3) Suppose that $D = \{(x,y) \mid x^2+y^2 \le 1\}$ and $f:\partial D \to \mathbb{R}$ is continuous. Show that the boundary value problem $\Delta u = u_{xx}+u_{yy}=0$ on D; $u=f$ on ∂D has at most one solution u(x,y).

4) Prove the special case of Lang's Theorem 3.3, p. 619, where everything takes place in the plane. Let $\sigma(x,y)=(u,v)$ be a continuously differentiable map from U 1-1 onto V, with continuously differentiable inverse. Here U and V are open sets in the plane. Let $\omega=f(u,v)du$ be a 1-form (with continuously differentiable f) on U. Show that $\sigma^* d\omega = d\sigma^*\omega$.

5) Show that if D is a nice oriented region in the plane (i.e., of the sort integrated over in Stokes Theorem). Assume D is contained in U. Let $\sigma(x,y)=(u,v)$ be a 1-1, onto continuously differentiable map $\sigma:U\to V$, with continuously differentiable inverse. Here U and V are open sets in the plane. Let $\omega = du \wedge dv$. Show that the formula

$$\iint_{\sigma(D)} \omega = \iint_D \sigma^*(\omega)$$

is the usual Jacobi change of variables formula in a double integral (Lang, p. 593).

6) Derive the Gauss Divergence Theorem from the general Stokes Theorem. The Gauss Theorem says

$$\iiint_D \left(f_x + g_y + h_z \right) dxdydz = \iint_{\partial D} (f,g,h) \bullet n \, dA .$$

Here n is the outward pointing normal from the surface ∂D and dA is the area element on the surface. You need to see that the right hand side can be written as

$$\iint_{\partial D} \left(fdydz + gdzdx + hdxdy \right) .$$

7) Show that the volume of a region R in 3-space is

$$vol(R) = \frac{1}{3} \iint_{\partial R} \left(xdydz + ydzdx + zdxdy \right) .$$

8) Find the volume of the wonderful torus from homework 2, problem 12, using problem 7).

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