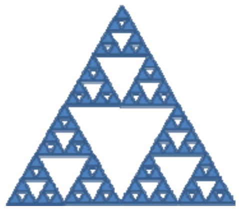
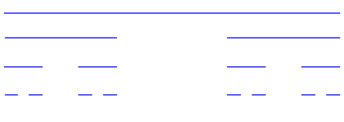


MATH 140c – TAKE - HOME FINAL – AUDREY TERRAS
Hand it in at my office 7408 AP&M before Fri. June 12, 12 – 3 P.M.
I will have office hours exam week: Mon.,Wed., Fri. from 12-3 pm.
No late papers will be accepted!

The rules: Choose 7 Problems to do out of the following 12.
You can work with other people and ask questions of Caleb and Audrey, and look at whatever books, websites (etc.) you want. But please write down on your exam who you worked with and what books, etc. you used. And please write up the answers to the exam in your own words.

1) Page 2 of fractals lecture. Show that the Cantor set can be expressed as certain triadic expansions and thus is uncountable.



2) Page 3 of fractals lecture. Show that the box dimension of the unit square is 2.

3) Page 5 of fractals lecture. Show that the box dimension of the set of rational numbers is 1. Compare with the box dimension of the Cantor set. How can it be that the Cantor set has a smaller box dimension than the rationals even though the Cantor set is uncountable?

4) Page 5 of fractals lecture. Show that the Sierpinski triangle is a self-similar set. Show using Thm. 2 that the box dimension of the Sierpinski triangle is ln3/ln2.

5) Answer the Why? in the Proof of Theorem 2 on p.5 of the fractals lecture.

6) Page 6 of fractals lecture. Show that any countable set M of real numbers has Lebesgue measure 0.

7) Page 6 of fractals lecture. Show that the Cantor C set (fractals lecture p. 2) has Lebesgue measure 0. Hint. To do this show that C_k in the definition of C has length $(2/3)^k$.

8) Page 6 of fractals lecture. Prove that the Devil's Staircase function f(x) is increasing, continuous and has derivative f'(x)=0 except on the Cantor set C, which has Lebesgue measure 0.

9) Page 8 of fractals lecture. Show that the Weierstrass function is a continuous function on [0,1].

$$f(t)=\sum_{k=1}^{\infty}\lambda^{(s-2)k}\sin(\lambda^k t)$$

10) Page 9 of fractals lecture. Fill in the details of the proof of Proposition 1, page 9 of the fractals lecture.

11) Page 10 of fractals lecture. Suppose $f:[a,b]\to\mathbb{R}$ has a continuous derivative. Show $\dim_B$graphf=1. Hint. Use the mean value theorem and Lemma 1.

12) Page 11 of fractals lecture. Show that any function satisfying condition 2 of Lemma 1 must be nowhere differentiable. It follows that the Weierstrass function is continuous but nowhere differentiable.