Math 142a – Homework #2 – due in class Wed. Jan 20 (or in drop box by 5 P.M. the same day).
Read Lang, *Undergraduate Analysis*, Chapter 0, Sections 4,5 plus Lang, Chapter 1, Sections 1,2,3 and Lecture Notes #1 & 2, pages 13-25.

The homework is worth a total of 100 points, with each problem worth the same amount.
Be sure and keep a homework notebook/folder.

1)  a) Explain why the set $\mathbb{Q}$ of rational numbers is denumerable.
    
    b) Suppose that $g:S \to T$ is 1-1 and onto. Show that $S$ is denumerable if and only if $T$ is denumerable.

2) Which of the following sets is denumerable:
    
    $\{2^n \mid n = 1,2,3,\ldots\}$ or the interval $[-2,1]$ ?

    Explain your answer.

3)  a) Suppose that $a$ is a real number and $n,m$ are positive integers. Prove that 
    
    $a^{n+m} = a^n \cdot a^m$.

    b) Prove the formula in part a) if both $n$ and $m$ are negative integers.

    c) Again suppose that $a$ is real and $n,m$ are positive integers. Show that 
    
    $(a^n)^m = a^{nm}$.

4)  a) Suppose that $a$ is a positive real number. Show that there is a positive integer $n$ such that 
    
    $\frac{1}{n} < x$.

    b) Show that $\sqrt{7}$ is irrational.

5)  a) Use the 2 order axioms ORD1 and ORD2 listed on p. 22 of the lectures plus the definition of $a < b$ to deduce that $x < y$ implies $x+z < y+z$ (which is IN3 in Lang, p. 22).

    b) Using the properties of inequalities (IN 1,2,3,4 on p. 22 of Lang) and the definition of absolute value, find the set of real numbers $x$ such that $|x^2 - 1| < \frac{1}{4}$. Draw a picture of the set.