Math 142b – Homework #6 - due in drop box Thurs. May 20
Read Lectures, Part 6, Read Lang, Chapter 10 to page 252.

For all of these problems, it should help to draw a picture.

1) Suppose $V$ is a normed vector space with norm denoted by $\| \cdot \|$. For any subset $A$ of $V$ define the closure of $A$ to be the set

$$\overline{A} = \{ x \in V \mid x \text{ is adherent to } A \}.$$ 

Show that if $A \subseteq B$, then $\overline{A} \subseteq B$.

2) Using the definition of closure of a set in problem 1, show that

$$\left\{ \left( x, y \right) \in \mathbb{R}^2 \mid x < y \right\} = \left\{ \left( x, y \right) \in \mathbb{R}^2 \mid x \leq y \right\}.$$ 

3) Let $f(x) = x^2$, for all $x \in [0, 1]$. Find a step function $s(x)$ on the interval $[0, 1]$ such that

$$\| f - s \|_\infty < 1/10.$$ 

4) Define the function $f(x) = x \sin(1/x)$, when $x \neq 0$, and $f(0) = 0$. Define $g(x) = 1$ if $x > 0$, $g(x) = -1$ if $x < 0$, and $g(0) = 0$. Show that $h(x) = g(f(x))$ is not the (uniform) limit of a sequence of step functions on the interval $[0, 1]$, using the norm $\| \cdot \|_\infty$. 