1) Define and give an example:
   a) norm;       b) scalar product;       c) equivalent norms;  
   d) limit of a sequence \( \{v_n\} \) in a normed vector space;   e)  Cauchy sequence in a normed vector space;  
   f) complete normed vector space;      g) adherent point to a set in a normed vector space; 
   h) \[ \lim_{x \to c} f(x) = L \] for \( c \) adherent to domain of \( f \) in a normed vector space;  
   i) continuous function \( f \) mapping a set in normed vector space \( V \) to another normed vector space;  
   j) uniform convergence of a sequence \( \{f_n\} \) of functions \( f_n \in C[a,b], \) the space of continuous real-valued functions on an interval \([a,b]\)  
   k) \( \| f \|_\infty, \| f \|_1, \| f \|_2 \) for \( f \in C[a,b] = \{ \text{continuous real valued functions on the interval } [a,b] \} \)  
   l) the cosine of the angle between 2 non-0 vectors \( v, w \) in a vector space \( V \) with a scalar product

2) True - False. Tell whether the following statements are true or false. Give a brief reason for your answer.  
   a) \( C[a,b] \) the space of continuous real-valued functions on an interval \([a,b]\), is a finite dimensional vector space.  
   b) For \( \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \in \mathbb{R}^2, \quad \| \mathbf{v} \| = v_1^2 + v_2^2 \) defines a norm on vectors in the plane.  
   c) The function \( f(x,y) = \frac{x^2}{x^2 + y^2} \), for \((x,y) \neq (0,0)\) and \( f(0,0)=0 \) is continuous on the plane \( \mathbb{R}^2 \).  
   d) Suppose that \( f_n \) is a sequence of continuous functions on the interval \([a,b]\). Suppose for every \( x \) in \([a,b]\) we have \( \lim_{n \to \infty} f_n(x) = f(x) \). Then \( f(x) \) is continuous on \([a,b]\).  
   e) \( \mathbb{R}^2 \) is a complete normed vector space.

3) State and prove the Cauchy-Schwarz inequality. What norm is being used in this inequality? Would it still be true if we replaced that norm with some other one?  

4) Suppose that \( \| \|_\alpha \) and \( \| \|_\beta \) are equivalent norms on a vector space \( V \). Show that if \( \{v_n\} \) is a sequence of vectors in \( V \) and \( L \in V \), we have  
   \[ \lim_{n \to \infty} \| v_n - L \|_\alpha \iff \lim_{n \to \infty} \| v_n - L \|_\beta . \]

5) Suppose that \( \langle v, w \rangle \) denotes a scalar product of 2 vectors \( v, w \) in the vector space \( V \). Show that \( \langle v, w \rangle \) is a continuous function of \( v \), holding \( w \) fixed. Is it uniformly continuous?
6) Suppose that \( L:\mathbb{R}^2 \to \mathbb{R}^2 \) is a linear map. Show that it is continuous.

7) Show that \( f:U \to W \), where \( U \) is a subset of a normed vector space \( V \) and \( W \) is a normed vector space, is continuous at a point \( a \in U \) if and only if for every sequence \( v_n \) in \( U \) such that \( \lim_{n \to \infty} v_n = a \) we have \( \lim_{n \to \infty} f(v_n) = f(a) \).

8) The function \( I \) mapping \( C[a,b] \), the space of continuous real valued functions on the interval \( [a,b] \) into \( \mathbb{R} \) defined by \( I(f) = \int_a^b f \) is continuous with respect to the \( \| \cdot \|_\infty \) norm on \( C[a,b] \) and the usual absolute value on \( \mathbb{R} \).

9) Show that the norms \( \| \cdot \|_\infty \) and \( \| \cdot \|_1 \) on the space \( C[a,b] \), the space of continuous real valued functions on the interval \( [a,b] \) into \( \mathbb{R} \) are not equivalent.

10) Explain how the picture below can be used to see the difference between \( \| f-g \|_\infty \) and \( \| f-g \|_1 \) assuming that \( f \) is the purple function which starts at the top and \( g \) is the blue function which starts at the bottom.