

**Math 109, Spring 2008 Midterm Exam #2 May 21, 2008**

**Instructor: Professor Chow**

**Instructions** Write clearly and neatly. Give complete proofs with a reasonable amount of detail. Most of the credit will be given for getting the main ideas, but points will be taken off for incorrect statements and incomplete proofs.

**NO CALCULATOR, CLOSED BOOK, CLOSED NOTES.**

**Your name** (4 points) \_\_\_\_\_

Problem	Type	Possible points	Your score
Name	Easy	4	
1	TBA	12	
2	TBA	12	
3	TBA	12	
4	TBA	12	
5	TBA	12	
6	TBA	12	
7	TBA	12	
8	TBA	12	
Total		100	

Facts that you may use include:

(A) Let  $a, b, c, m,$  and  $n$  be integers. If  $a$  divides each of  $b$  and  $c$ , then  $a$  divides  $nb + mc$ .

(B) Let  $a$  and  $b$  be nonzero integers and let  $d = \gcd(a, b)$ . Then there exist integers  $m$  and  $n$  such that

$$d = ma + nb.$$

(C) Let  $a$  and  $b$  be nonzero integers. The number  $d = \gcd(a, b)$  is the smallest number in the set

$$S = \{ma + nb : m, n \in \mathbb{Z}\} \cap \mathbb{N}.$$

(D) Let  $p$  be a prime and let  $a$  and  $b$  be integers. If  $p|ab$ , then  $p|a$  or  $p|b$ .