Background information about #43. (Not necessary to do the problem.) We will go into more detail later in the course.

i.e. means 'that is'

$\doteqdot$ means defined to be equal to

$\mathbb{N} \doteqdot \{1, 2, 3, \ldots\}$ the set of natural numbers (i.e., positive integers)

The following definition is on p. 244 (§8.3) of Fletcher and Patty.

A **group** is a set G with a binary operation $\cdot$ which assigns to each a pair of elements a and b of G an element $a \cdot b$ of G, where $\cdot$ has the following properties:

(a) (associative law)
$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$
for all $a, b, c \in G$.

(b) (identity element) there exists an element e of G such that
$$a \cdot e = e \cdot a = a$$
for every $a \in G$.

(c) (inverse of an element) for each $a \in G$ there exists $x \in G$ such that
$$a \cdot x = x \cdot a = e.$$
We denote the inverse of a by a^{-1}.

If $a \in \mathbb{N}$, then the powers of a are denoted by
$$a^n \doteqdot a \cdots a \text{ (where } a \text{ occurs } n \text{ times).}$$

Examples of groups:

1. $\mathbb{Z}$ means the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. The integers with addition $(\mathbb{Z}, +)$ form a group.
2. $\mathbb{R}$ means the set of real numbers. The real numbers with addition $(\mathbb{R}, +)$ form a group.
3. The real numbers with multiplication $(\mathbb{R}, \cdot)$ is not a group since 0 does not have an inverse.
4. The nonzero real numbers with multiplication $(\mathbb{R} \setminus \{0\}, \cdot)$ is a group.

We also define

1. $a^0 \doteqdot e$
2. for n a negative integer,
$$a^n \doteqdot \left(a^{-n}\right)^{-1}.$$
Note that $-n \in \mathbb{N}$ so that a^{-n} denotes a multiplied by itself with $-n$ number of a's and $(a^{-n})^{-1}$ denotes the inverse of a^{-n}.