

Math 109, Winter 2003 Midterm Exam #1 January 31, 2003

Instructor: Professor Chow

Instructions Write clearly and neatly. Give complete proofs with a reasonable amount of detail. Most of the credit will be given for getting the main ideas, but points will be taken off for incorrect statements and incomplete proofs.

NO CALCULATOR.

CLOSED BOOK, CLOSED NOTES.

1. (21 points total)

- (a) (10 points) Fill in the **truth table** for P , Q , $\neg Q$, $P \rightarrow Q$, $\neg(P \rightarrow Q)$ and $P \wedge \neg Q$.

P	Q	$\neg Q$	$P \rightarrow Q$	$\neg(P \rightarrow Q)$	$P \wedge \neg Q$

- (b) (3 points) Is $\neg(P \rightarrow Q) \leftrightarrow P \wedge \neg Q$ a **tautology**? Why or why not?
(c) (8 points) [Unrelated to first two parts.] Is the following statement **true** or **false**?

$$(\forall y)(\exists x)(x > y^2 + 1).$$

Give a short proof.

2. (13 points) Let a and L be real numbers and f be a function. State the **negation** of the following proposition without using any negative words. For every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $|x - a| < \delta$ and $x \neq a$, then $|f(x) - L| < \varepsilon$.
3. (14 points total)
- (a) (7 points) Prove that if b is an integer such that b^3 is even, then b is even.
- (b) (7 points) Let a and b be positive integers such that $a^2 = b^3$. Prove that if 4 divides a , then 4 divides b .
4. (13 points) Prove that if $n \geq 6$ and $n = ab$, where $1 < a, b < n$, then n divides $(n - 1)!$. Consider the cases where $a \neq b$ and $a = b$ separately.
5. (13 points) Prove that if $n = 3k + 1$ for some integer k or $n = 3\ell + 2$ for some integer ℓ , then n^2 is not divisible by 3.
6. (13 points) Prove that if X and Y are sets such that $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$, then $X \subseteq Y$. ($\mathcal{P}(X)$ denotes the power set of X .)
7. (13 points) Prove that $A - (B - C) \subseteq A \cap (B' \cup C)$.