Section 1.4: The Tangent Problem

1. Let \( f \) be the function defined by \( f(x) = 4x^2 \). Let \( x \) be different from 3. What is the slope \( m_x \) of the line through the points \((3, 36)\) and \((x, 4x^2)\)? Simplify your answer as much as possible.

2. Let \( f \) be the function defined by \( f(x) = \frac{2}{3x} \). Let \( x \) be different from 0 and 1. What is the slope \( m_x \) of the line through the points \((1, \frac{2}{3})\) and \((x, \frac{2}{3x})\)? Simplify your answer as much as possible.

3. The point \( P(2, -1) \) lies on the curve \( y = \frac{1}{1-x} \).
   (a) If \( Q \) is the point \((x, \frac{1}{1-x})\), use your calculator to find the slope of the secant line \( PQ \) for the following values of \( x \): 1.5, 1.9, 1.99, 2.5, 2.1, 2.01, 2.001.
   (b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at \( P(2, -1) \).
   (c) Using the slope from part (b), find an equation of the tangent line to the curve at \( P(2, -1) \).

4. Let \( f \) be the function defined by \( f(x) = -\frac{1}{x^2} \). Let \( x \) be different from 0 and 2.
   (a) What is the slope \( m_x \) of the line through the points \((2, -\frac{1}{4})\) and \((x, -\frac{1}{x^2})\)? Simplify your answer as much as possible.
   (b) Guess the value of \( \lim_{x \to 2} m_x \), and determine an equation for the line tangent to the graph of \( f \) at \((2, -\frac{1}{4})\).

Section 1.5: The Limit of a Function

1. Use the given graph of \( f \) (see Figure 1) to state the value of each quantity, if it exists. If it does not exist, explain why.
   (a) \( \lim_{x \to 2^-} f(x) \); (b) \( \lim_{x \to 2^+} f(x) \); (c) \( \lim_{x \to 2} f(x) \); (d) \( f(2) \); (e) \( \lim_{x \to 4} f(x) \); (f) \( f(4) \).

2. For the function \( g \) whose graph is given (see Figure 2), state the value of each quantity, if it exists. If it does not exist, explain why.
   (a) \( \lim_{x \to 0^-} g(t) \); (b) \( \lim_{x \to 0^+} g(t) \); (c) \( \lim_{x \to 0} g(t) \); (d) \( \lim_{x \to 2^-} g(t) \); (e) \( \lim_{x \to 2^+} g(t) \); (f) \( \lim_{x \to 2} g(t) \);
   (g) \( g(2) \); (h) \( \lim_{x \to 4} g(t) \).