9 March 1999
revised 23 May 1999

WHY QUANTUM STRATEGIES ARE QUANTUM MECHANICAL

David A. Meyer

Project in Geometry and Physics
Department of Mathematics
University of California/San Diego
La Jolla, CA 92093-0112
dmeyer@chonji.ucsd.edu

and Center for Social Computation/Institute for Physical Sciences
Los Alamos, NM

ABSTRACT

Winning strategies in PQ PENNY FLIP are quantum mechanical in exactly the way quantum algorithms are.

1998 Physics and Astronomy Classification Scheme: 03.67.Lx, 03.67.-a, 02.50.Le.
American Mathematical Society Subject Classification: 81P15, 90D05.
Journal of Economic Literature Classification System: C72.
Key Words: quantum computation; game theory; hidden variable theories

Van Enk's observation that there are classical models for Q's strategy in PQ PENNY FLIP [1] is, of course, correct; but this does not mean that "there is nothing quantum-mechanical about that strategy" [2]. One might equally well say that there is nothing classical about Picard's pure strategies since there are quantum models for flipping (or not) a two-state system [3]. Clearly, the expansion of Q's strategy set which allows him to win every game can be realized in either a quantum or a classical system, but to argue that "A single qubit is not a truly quantum system" because it can be "mocked up by a classical hidden-variable model" [2] is, as Heisenberg put it, to "attempt to put new wine into old bottles. Such attempts are always distressing, for they mislead us into continually occupying ourselves with the inevitable cracks in the old bottles, instead of rejoicing over the new wine." [4]

Nevertheless, since van Enk suggests that we should put PQ PENNY FLIP into an old bottle, let us identify the cracks. Our present interest is less in ruling out classical hidden-variables models for quantum mechanics and more in demonstrating computational advantages for quantum over classical systems. From this perspective models should be dynamical, require only poly(log) precision specification of operations [5], and most importantly, scale up as the number of Hilbert space factors (*e.g.*, qubits) increases. The Deutsch-Jozsa [6], Simon [7] and Grover [8] algorithms, each of which is structured as a PQ game [1], describe quantum computations for which *any* classical model—including ones like those suggested by van Enk [2]—must scale badly: exponentially in the first two cases and quadratically in the third. The old bottles can hold only a few drops of new wine—any more leaks out through the cracks.

Although van Enk alludes to entanglement when he mentions the Bell inequalities [9], in fact, entanglement of intermediate states is not even necessary for quantum algorithms to outperform classical ones: Imagine Picard and Q playing a two qubit game initialized at $|00\rangle$, where Picard is constrained to make one of four moves—corresponding to the possible maps $f : \mathbb{Z}_2 \to \mathbb{Z}_2$ *via* $|x, y\rangle \to |x, y \oplus f(x)\rangle$ for $x, y \in \mathbb{Z}_2$, a basis for $\mathbb{C}^2$, and where $\oplus$ denotes addition mod 2. If Q's objective is to identify Picard's choice of function as surjective or not at the end of the game, no classical strategy can ensure he wins more than half the time. But the simple improvement on the one bit Deutsch-Jozsa [6] and Simon [7] algorithms consisting of Q first acting by $H \otimes H\sigma_z$ and last by $H \otimes \mathbf{1}_2$ (where $H = \left(\begin{smallmatrix}1 & 1\\ 1 & -1\end{smallmatrix}\right)/\sqrt{2}$ is the Hadamard transform) guarantees a win with probability 1 since the first qubit is $|1\rangle$ when Picard's choice is surjective and $|0\rangle$ otherwise. *At no turn in the game are the two qubits entangled; Q's strategy works by a clever interference of amplitudes just as it does in* PQ PENNY FLIP [1]. There the amplitudes for the computational paths terminating at tails (T) cancel independently of Picard's move: For example, suppose Picard flips the penny on his turn. Then the four computational histories in the $\{H, T\}$ basis are $HHTT$ and $HTHT$, with cancelling amplitudes of $-1/2$ and $1/2$, respectively, and $HHTH$ and $HTHH$, each with amplitude $1/2$, reinforcing to give heads (H) with probability 1 at the end of the game.

The relevance of classical models for quantum systems depends upon the use to which they are put. In the modern context of quantum information processing, models must scale with the number of qubits, and be dynamical. Thus, despite the fact that there is a classical model for a single qubit, it is most useful to consider the simple quantum strategy illustrated in PQ PENNY FLIP [1] as quantum mechanical. Even a single drop holds the taste of a new wine.

Acknowledgements

I thank Chris Fuchs and Raymond Laflamme for useful discussions. This work has been partially supported by Microsoft Research and by ARO grant DAAG55-98-1-0376.

References and notes

[1] D. A. Meyer, "Quantum strategies", *Phys. Rev. Lett.* **82** (1999) 1052–1055.
[2] S. J. van Enk, "On quantum and classical game strategies", preprint (1999).
[3] Let me take this opportunity to correct a minor substantive error in [1]. The single qubit error correcting code discussed at the end of the paper must be used only to encode a classical bit.
[4] W. Heisenberg, "The development of the interpretation of the quantum theory", in W. Pauli, ed., with the assistance of L. Rosenfeld and V. Weisskopf, *Niels Bohr and the Development of Physics* (New York: Pergamon Press 1955) 12–29.
[5] P. W. Shor, "Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer", *SIAM J. Comput.* **26** (1997) 1484–1509;
M. H. Freedman, "Topological views on computational complexity", *Proceedings of the International Congress of Mathematicians*, Vol. II (Berlin, 1998), *Doc. Math. J. DMV*, Extra Vol. ICM II (1998) 453–464;
D. A. Meyer, "Finite precision measurement nullifies the Kochen-Specker theorem", UCSD preprint (1999).
[6] D. Deutsch and R. Jozsa, "Rapid solution of problems by quantum computation", *Proc. Roy. Soc. Lond. A* **439** (1992) 553–558.
[7] D. R. Simon, "On the power of quantum computation", in S. Goldwasser, ed., *Proceedings of the 35th Symposium on Foundations of Computer Science*, Santa Fe, NM, 20–22 November 1994 (Los Alamitos, CA: IEEE Computer Society Press 1994) 116–123.
[8] L. K. Grover, "A fast quantum mechanical algorithm for database search", in *Proceedings of the 28th Annual ACM Symposium on the Theory of Computing*, Philadelphia, PA, 22–24 May 1996 (New York: ACM 1996) 212–219.
[9] J. S. Bell, "On the Einstein-Podolsky-Rosen paradox", *Physics* **1** (1964) 195–200.