This is the second midterm from the last time I taught this class. You may find it a bit difficult, but it will be good practice to try to do all the problems.

1.a. [18 points] Prove that \( \sum_{i=1}^{n} (3i^2 - 3i + 1) = n^3. \)

b. [7 points] Draw a picture that illustrates why this formula is true.

2. [25 points] For \( n \in \mathbb{N} \), let \( f_n(x) = f'_{n-1}(x) \), where \( f'_{n-1}(x) \) means the derivative with respect to \( x \) of \( f_{n-1}(x) \); and let \( f_0(x) = \cos x \). Prove that for all \( k \in \mathbb{N} \), \( f_{2k}(x) = (-1)^k \cos x \).

3.a. [7 points] Sketch the line \( y = -\frac{2}{5}x + \frac{2}{3} \).

b. [18 points] Prove that this line does not contain any point \((x, y)\) where \( x \) and \( y \) are both integers.

4. [25 points] Let \( x, y, z, r \in \mathbb{N} \) and let \( r \) be odd. Prove that if \( x^2 + y^2 + z^2 = r^2 \), then exactly one of \( x, y, z \) is odd. [Hint: First show that for any \( n \in \mathbb{N} \), either \( 4|n^2 \) or the remainder when \( n^2 \) is divided by 4 is 1.]