MATH 109 Midterm 2 12 November 2003

This is the second midterm from the last time I taught this class. You may find it a bit difficult, but it will be good practice to try to do all the problems.

1.a. [18 points] Prove that $\displaystyle\sum_{i=1}^{n}(3i^2 - 3i + 1) = n^3$.

b. [7 points] Draw a picture that illustrates why this formula is true.

2. [25 points] For $n \in \mathbb{N}$, let $f_n(x) = f'_{n-1}(x)$, where $f'_{n-1}(x)$ means the derivative with respect to x of $f_{n-1}(x)$; and let $f_0(x) = \cos x$. Prove that for all $k \in \mathbb{N}$, $f_{2k}(x) = (-1)^k \cos x$.

3.a. [7 points] Sketch the line $y = -\dfrac{2}{5}x + \dfrac{2}{3}$.

b. [18 points] Prove that this line does not contain any point (x, y) where x and y are both integers.

4. [25 points] Let $x, y, z, r \in \mathbb{N}$ and let r be odd. Prove that if $x^2+y^2+z^2 = r^2$, then exactly one of x, y, z is odd. [Hint: First show that for any $n \in \mathbb{N}$, either $4|n^2$ or the remainder when n^2 is divided by 4 is 1.]