109 Lecture A MIDTERM 1

Each problem is worth 25 points. Please be specific in your answer to problem 1, and please write out your proofs for the propositions in problems 2, 3 and 4 in complete sentences, justifying how you get from step to step.

1. Consider the proposition: “Yao Ming is tall.” Explain why the following attempted proof of this proposition is wrong:

   Tall basketball players can easily dunk.
   Yao Ming can easily dunk.
   Yao Ming is a basketball player.
   Therefore Yao Ming is tall.

2. Recall that for \( a, b \in \mathbb{Z} \), \( a|b \) means that there is an integer \( q \) such that \( b = aq \). Prove that the relation \( | \) is transitive.

3. Prove that there is no pair of real numbers \( (x, y) \) that solves the following pair of equations:

   \[
   x^2 + y^2 = 1 \\
   y = 3.
   \]

4. Prove that \( \sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \) for integers \( n \geq 0 \).