15 October 2013

INTRODUCTION TO MATHEMATICAL MODELING SCHOOL CHOICE

David A. Meyer

Department of Mathematics
University of California/San Diego
La Jolla, CA 92093-0112
http://math.ucsd.edu/~dmeyer/; dmeyer@math.ucsd.edu

Motivation

Curie Elementary School in the San Diego Unified School District (SDUSD) had a teacher declared "excess" [1] in the fifth week of this school year, because the enrollment in grades K through 3 was 10 less than the budget-mandated 27 times the number of teachers. A teacher was removed, and the students in one of the third grade classes were distributed to the the remaining 3 classes.

Why did this happen? Can/should it be avoided? We'd like to build a mathematical model to try to answer these questions.

School choice in the SDUSD

Students who live in the local neighborhood of a school have the option of attending that school. In addition, students from outside the neighborhood can enter a lottery for a place at that school, where the number of places is determined before the lottery, which takes place in the spring, about 4 months before the school year starts.* Presumably the number of students allowed to "choice in" to a school is determined so as to get the number of students per teacher as close to 27 as possible. Let's try to understand how hard it might be to do this, and whether we should expect errors of underenrollment, or of overenrollment.

Estimating the number of students from the neighborhood

In class we identified many factors that could affect the number of students from the neighborhood who choose to enroll in the local school:

* This is a simplification. There are various categories of students from outside the neighborhood which have different priorities for "choicing in" to the school [2], but these details won't affect the way we conceptualize a model.

· the number of children in the neighborhood;
· the quality of the school (test scores, teachers, safety, culture), and of other schools;
· special programs at the school, or at other schools;
· size of the neighborhood (since that affects travel time to the school);
· capacity of the school, *i.e.*, is it overcrowded?;
· hours of operation, and after school programs;
· the overall state of the economy, insofar as it might affect families' ability to transport students to another school, or pay for private school;
· whether some students have relatives currently or previously at the school;

but the most useful single piece of information for predicting the number of local students who will enroll in grade g this year seems likely to be the number of local students who were enrolled in grade $g-1$ last year. Combining some of this information, and maybe some we didn't consider, is the job of the SDUSD Demographer, who predicts the number of students from the neighborhood who will enroll in each grade of each school each year.

Clearly this is a difficult task, and impossible to get exactly right. Without access to all these pieces of information, let us assume that they combine to produce a number n of students who might enroll, and a probability p that each of them, independently, does so. With this assumption, the number of local students who enroll is a random variable X with a binomial probability distribution:

$$\text{Prob}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}.$$

Recall that $\mathsf{E}[X] = pn$ and $\mathsf{Var}[X] = np(1-p)$. Most generally, there should be some probability distribution, with its expectation value and variance.

School capacity

Suppose the capacity of the school is $K > np$, *i.e.*, the school can handle more than the local students who are expected to enroll. In this case it seems to be the policy of the SDUSD to allow some students from outside the neighborhood to choice in to the school [2]. To determine how many choice students to allow, there must be some precise definition of "capacity".

I spoke to several people in the SDUSD administration but was unable to get anyone to define "capacity". In practice, judging by my understanding of the way things happened at Curie Elementary, there are some teachers who are somehow attached to a school, and in many cases to a specific grade at the school. Let us call the number of such teachers t, where for third grade at Curie Elementary, $t = 4$.

There is also a annual budget for the SDUSD, which specifies the number of students per class. Roughly speaking, this number is as small as the District can afford, *i.e.*, they have a good idea of the total number of students in the District, and a specified number of teachers whose salaries are included in the budget. Dividing the former by the latter gives the class size, s.* For the purposes of doing explicit calculations, let $s = 27$, which is the budget-mandated elementary school class size for this year [3].

Absent an official and precise definition of capacity, we will take it to be the class size, multiplied by the number of teachers, $K = st$.

Utility

From the point of view of the SDUSD, given the constraints under which they are working, it is optimal to have exactly s students in each class.† Having more, or fewer, students must be suboptimal, *i.e.*, those situations have lower utility, *for the District*. To quantify this, let $u(e)$ be the utility the SDUSD obtains from having an enrollment of e students in a grade at a school. Let $u(K) = 0$; then $u(e)$ will be negative for $e > K$ and for $e < K$.

First c,onsider over enrollment, $e > K$. While the students (and their parents) might object to classes with large sizes, the SDUSD's utility seems likely to be determined by how well the students do on the standardized tests that are used to evaluate how well an individual school, and the whole District, is performing. There is a substantial, and inconsistent, literature on the effect of class size on student performance, but it seems pretty reasonable to me to expect student performance to decrease with increasing class size. A plausible model for this is that how much a student learns depends positively on how much attention his/her teacher pays to him/her; since the teacher has a finite amount of attention, if it is split evenly, then how well a student does should be proportional to t/e.‡ This implies

$$u(e) = -P\Big(\frac{t}{K} - \frac{t}{e}\Big), \quad \text{for } e \geq K.$$

Here we subtract from t/K so that $u(K) = 0$, and P is a proportionality constant that depends on the penalty a school, and hence the SDUSD, suffers for not having sufficiently good average student performance. This is a simplification, since schools are assessed on both absolute performance, and on improvement [4], but it seems to capture the essence of the loss of utility associated with over enrollment.

Second, consider under enrollment, $e < K$. The most obvious thing to say is that in this case the SDUSD is wasting money by paying a teacher to teach fewer students, so the disutility is an amount proportional to the number of students less than capacity:

$$u(e) = -M(K - e), \quad \text{for } e \leq K.$$

We'll consider other possibilities after we examine the consequences of this utility function.

---

* Of course, class sizes are not uniform: underperforming schools are allowed to have smaller class sizes, elementary school classes are smaller than high school classes, and often kindergarten classes are smaller still.

† With the caveats in the previous footnote.

‡ A reasonable argument can be made that attention is not split evenly, and thus student performance might decay faster than inversely with e.

References

[1] M. Magee, “Positions cut, teachers to vie for jobs”, *San Diego Union-Tribune* (13 October 2013); http://www.utsandiego.com/news/2013/oct/13/excessed-teachers-compete-jobs/.

[2] SDUSD, “School Choice/Open Enrollment Act”, http://www.sandi.net/site/default.aspx?PageID=915.

[3] M. Magee, “Details emerge in SDUSD’s job-saving budget”, *San Diego Union-Tribune* (25 February 2013); http://www.utsandiego.com/news/2013/Feb/25/tp-details-emerge-in-sdusds-job-saving-budget/.

[4] *Public Schools Accountability Act*; http://www.cde.ca.gov/ta/ac/pa/.