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INTRODUCTION TO MATHEMATICAL MODELLING LECTURE 1: RE-INTRODUCTION

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Introduction

This is the second quarter of a two quarter course on mathematical modelling. Students who did not take the first quarter are welcome, but will need to do a little extra work to catch up. The goals for all of us in this course are:

> to appreciate that mathematical models can be constructed for many real-world systems, not just in physics, chemistry and engineering, and to construct some;
>
> to learn some general purpose modelling techniques in the context of specific problems; and
>
> to understand how such problems can lead to the development of new mathematical ideas, and even motivate proving theorems.

Prerequisites

The prerequisites for this course are calculus and either linear algebra or ordinary differential equations. I will explain parts of probability and statistics, geometry, analysis, and combinatorics as we need them; some of this material may be new to some students. It will be useful to be fluent in Matlab or Mathematica, or in some general purpose programming language.

Requirements

I expect interest and enthusiasm from the students in this class. Grades will be determined as follows:

[40%] class participation—asking questions, contributing to the discussion, describing homework solutions, presenting papers

[30%] written homework solutions, assigned periodically

[30%] final written report on a project to be agreed on with me

The zeroth homework assignment, particularly for the new students, but possibly a useful review for everyone, is:

Homework: Read the notes from last quarter that are on-line at:
http://math.ucsd.edu/~dmeyer/teaching/111winter04.html,
and come to class on Friday with questions.

To help review, I'm going to tell you about my Spring Vacation. I spent 17 days in China, in Beijing, Xi'an, Guilin, Chongqing, Yichang, Shanghai and Suzhou. Despite still being a "communist" country, China is moving towards a market economy. No longer is it bad to be rich, as it certainly was during the Cultural Revolution, and in principle has been since the Communist Revolution in 1949. Many of the people we met were obsessed with money, or at least more verbal about it than is common in ordinary conversation even in the U. S. (where, for many people, success is measured by money accumulated). As one person explained to us, "In English, saying 'money' makes you frown, but in Mandarin, saying '錢' ('qián') makes you smile!" In fact, while we were there, the National People's Congress approved a constitutional amendment protecting private property rights (although as far as I can tell, not to land).

All of this suggests investigating the distribution of wealth among individuals in China. In a truly communist society, it should be approximately uniform, but it appears that things have changed: Figure 1.1 is a histogram of the wealths of the richest 100 people in China in 2003, according to *Forbes* [1]; Figure 1.2 shows the same data, but plotted in the form of wealth w as a function of rank r (first richest, second richest, *etc.*). Notice that the graph in Figure 1.2 looks something like a hyperbola, *i.e.*, $w = k/r$ for some constant k. Taking the logarithm of this equation gives $\log w = \log k - \log r$, which means that if the data did lie on a hyperbola and they were plotted on a log-log plot, they would lie on a straight line with slope -1.

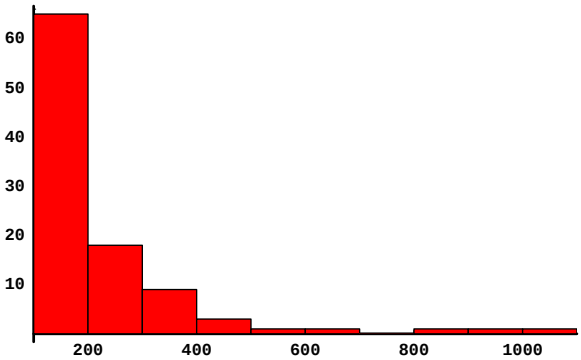

Figure 1.1. Histogram of the wealths of the 100 richest people in China in 2003, in US$M.

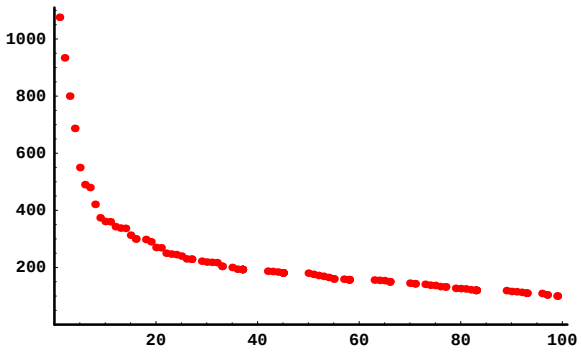

Figure 1.2. The same data, with wealth plotted as a function of rank.

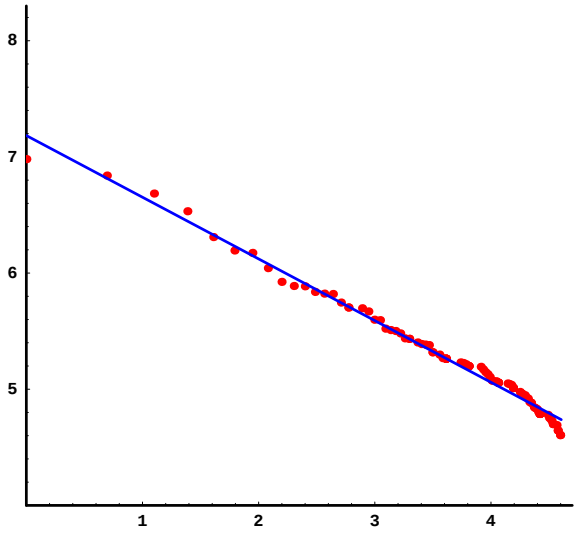

Figure 1.3. Log-log plot of wealth as a function of rank for the 100 richest people in China in 2003.

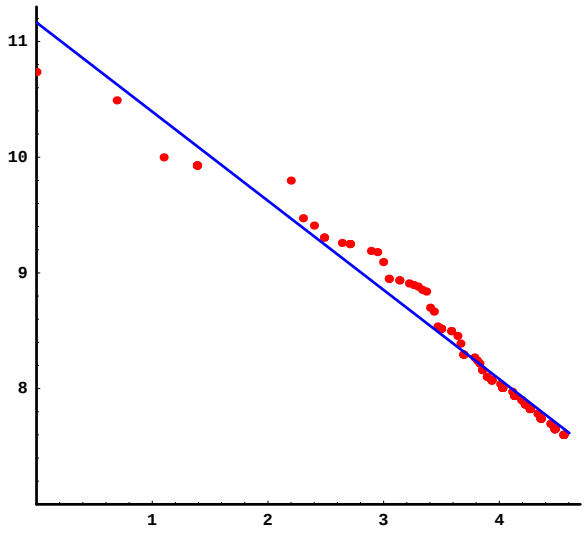

Figure 1.4. Log-log plot of wealth as a function of rank for the 100 richest people in the U. S. in 2003.

Figure 1.3 shows the log-log plot of wealth as a function of rank; it is close to being linear, with a least-squares fit giving a slope not of -1, but of approximately -0.53. (In general, we call functions $f(x) = kx^{-\alpha}$, *power laws*.) This means that the distribution of wealth in China is far from being uniform, although these data alone tell us nothing about the wealths of the remaining 1.3 billion or so people in China. For comparison, Figure 1.4 shows the log-log plot of wealth as a function of rank for the richest 100 people in the U. S. in 2003, again according to *Forbes* [2]. As we saw last quarter [3], this plot is also close to being linear, with a least-squares fit giving a slope of approximately -0.77. That is, the wealth distribution in the U. S. is *less equal* than in China, at least among the richest 100 people. For the U. S. *Forbes* actually lists the richest 400 people, and the fit to this data is very close to the fit to the richest 100 [3].

Homework: Find data for the distribution of wealth across as much of the population in China, and in the U. S., as possible. How far does the power law distribution for the richest people extend?

Another aspect of China that was impossible to miss is the pace of change. Not only are the laws changing, as I mentioned above, but there is construction everywhere: buildings, roads, bridges and dams. Industrial development has polluted the water and the air [4]; nowhere was the water drinkable, and when we arrived at the Los Angeles airport on saturday, our first reaction was "Wow, the air is so clean!".

This suggests that the distribution of wealth may also be changing rapidly. *Forbes* has been publishing their list of richest people in China since 1999, and including estimates of their wealths since 2000. Figure 1.5 shows the log-log plots and linear fits for the data for these four years. Perhaps surprisingly, the absolute value of the slope (the power-law expo-

nent), which is a monotonic measure of wealth inequality, has been *decreasing* for these four years: 0.94, 0.73, 0.59, 0.53. Of course, this could be a consequence of the global collapse of the stock market in 2000. To interpret these observations, therefore, we should compare them to the corresponding data for some other countries.

Homework: Plot historical data from the *Forbes* 400 lists of the richest people in the U. S. Are there trends in wealth inequality as measured by the slope of the least-squares fit to the log-log plot of wealth as a function of rank? How does the trend for 2000–2003 compare with the trend in China? Email me your results and we will discuss them in class on Monday 5 April.

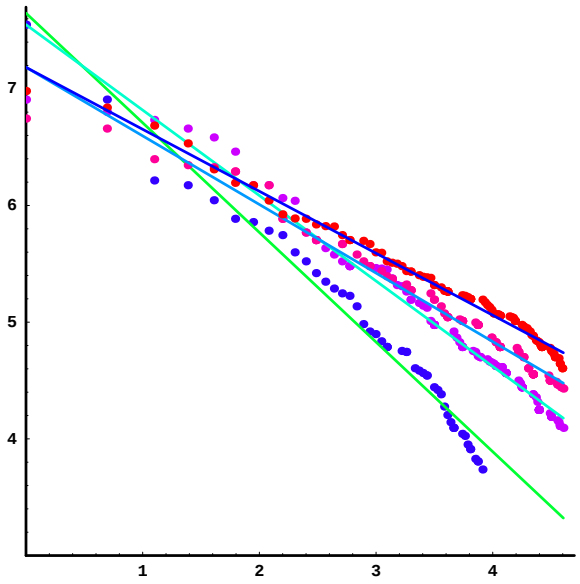

Figure 1.5. Log-log plots of wealth as a function of rank for the richest people in China in the years 2000 (blue), 2001 (purple), 2002 (magenta) and 2003 (red). The corresponding least-squares fits are shown in green, cyan, light blue, and blue, respectively.

Last quarter we constructed models for wealth distribution [5], but we did not really identify what parameters control the exponent in the cases where we obtained a power-law distribution.

Homework: Investigate models for wealth distribution that produce power-law distributions for wealth at the richest end of the distribution. Determine what parameters in the models control the exponent. Can you interpret these parameters in terms of real-world quantities? This is a possible project for the quarter.

References

[1] "China's 100 richest", *Forbes* (30 October 2003); http://www.forbes.com/2003/10/29/chinaland.html.
[2] "The Forbes 400", *Forbes* (18 September 2003); http://www.forbes.com/richlist2003/rich400land.html.
[3] D. A. Meyer, "Introduction to Mathematical Modelling: Other distributions", http://math.ucsd.edu/~dmeyer/teaching/111winter04/IMM040114.pdf.
[4] J. Becker and B. Sacha, "China's growing pains", *National Geographic* (March 2004); http://magma.nationalgeographic.com/ngm/0403/feature4/index.html.
[5] D. A. Meyer, "Introduction to Mathematical Modelling: Modelling wealth distribution", http://math.ucsd.edu/~dmeyer/teaching/111winter04/IMM040123.pdf.